UTA PhD Diagnosis Exam (Spring 2013)

Digital Signal Processing

Instructions:
• Verify that your exam contains 7 pages (including the cover sheet).
• Please be sure to use blank paper to write your answers. If more space is needed, please ask the instructor for extra paper. DO NOT WRITE ON THE BACK OF A SHEET!
• The point values listed on this exam serve only as a guideline. The Dept reserves the right to make modifications to the weighting of the problems.
• Calculator is okay.

I Choose to work on Problems _____ and _______ (Choose only 2 from the 3 problems).

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Problem 1: [50 pts]

A filter is needed to recover \( x_1(n) \) from the signal, \( x(n) = x_1(n) + x_2(n) + x_3(n) \), where \( x_1(n) = \sin(0.7n) \), \( x_2(n) = \sin(1.5n) \), and \( x_3(n) = \cos(2.7n) \).

(a) [10 pts] What kind of frequency selective filter is required, lowpass, highpass, bandpass, or bandreject?

(b) [15 pts] Specify ranges for the filter’s cut-off frequency or frequencies \( w_i \) in radians, for \( i = 1,2,\ldots \). (There is either 1 cut-off frequency, or 2)

(c) [15 pts] Assuming that the filter’s frequency response \( H(e^{jw}) \) is real, find the impulse response \( h(n) \) using the inverse discrete time Fourier transform.

(d) [10 pts] We want to convert \( h(n) \) to be a causal finite impulse response (FIR) filter \( h_1(n) \), so that it can be applied in real time. If the filter is to be nonzero for \( 0 \leq n \leq N-1 \), express \( h_1(n) \) in terms of the symbols \( h() \) and \( N \).

(Hint: Consider the appropriate time delay)

Inverse DTFT:  
\[
h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{jw}) e^{jwn} \, dw
\]
Problem 2: [50 pts]
Find \( y(n) \) in terms of \( x(n) \) and \( h(n) \) (which can be complex) if

(a) [25 pts] \( Y(k) = H^*(-k)_N \cdot X(k) \)

(b) [25 pts] \( Y(k) = \sum_{m=0}^{N-1} H(m + k)_N \cdot X(m + 2k)_N \)

Remember that the Forward and Inverse discrete Fourier transforms (DFTs) are defined as

\[
Y(k) = \sum_{n=0}^{N-1} y(n) \cdot W_n^{nk}, \quad y(n) = \frac{1}{N} \sum_{k=0}^{N-1} Y(k) \cdot W_n^{-nk},
\]

\[
W_N = \exp\left(-j \frac{2\pi}{N}\right)
\]
Problem 3: [50 pts]
Assume that

\[ X(e^{jw}) = e^{-4|w|} \quad \text{for} \quad |w| \leq \pi \]

(a) [15 pts] Find \( x(0) \)
(b) [15 pts] Using Parseval's equation, find the numerical value of

\[ \sum_{n=-\infty}^{\infty} x^2(n) \]

(c) [10 pts] Given your answer in part (b), give the limit of \( x(n) \) as \( n \) approaches infinity.
(d) [10 pts] Find a real expression for \( x(n) \).

Parseval's Equation:

\[ \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| X(e^{jw}) \right|^2 dw \]