Money and Banking

ECON3303

Lecture 4: Understanding Interest Rates

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Measuring Interest Rates

• Present Value:
• A dollar paid to you one year from now is less valuable than a dollar paid to you today
• Why?
  – A dollar deposited today can earn interest and become $1 \times (1+i)$ one year from today.
Discounting the Future

Let $i = .10$

In one year  $100 \times (1 + 0.10) = 110$

In two years  $110 \times (1 + 0.10) = 121$

or $100 \times (1 + 0.10)^2$

In three years  $121 \times (1 + 0.10) = 133$

or $100 \times (1 + 0.10)^3$

In $n$ years

$100 \times (1 + i)^n$
Simple Present Value

PV = today's (present) value
CF = future cash flow (payment)
i = the interest rate

$$PV = \frac{CF}{(1 + i)^n}$$
• Cannot directly compare payments scheduled in different points in the time line
Four Types of Credit Market Instruments

- Simple Loan
- Fixed Payment Loan
- Coupon Bond
- Discount Bond
Yield to Maturity

• The interest rate that equates the present value of cash flow payments received from a debt instrument with its value today
Simple Loan

PV = amount borrowed = $100
CF = cash flow in one year = $110

\[ n = \text{number of years} = 1 \]

\[ \$100 = \frac{\$110}{(1 + i)^1} \]

\[ (1 + i) \times \$100 = \$110 \]

\[ (1 + i) = \frac{\$110}{\$100} \]

\[ i = 0.10 = 10\% \]

For simple loans, the simple interest rate equals the yield to maturity.
Fixed Payment Loan

The same cash flow payment every period throughout the life of the loan

LV = loan value

FP = fixed yearly payment

\( n = \text{number of years until maturity} \)

\[
LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \cdots + \frac{FP}{(1+i)^n}
\]
Coupon Bond

Using the same strategy used for the fixed-payment loan:

\[ P = \text{price of coupon bond} \]

\[ C = \text{yearly coupon payment} \]

\[ F = \text{face value of the bond} \]

\[ n = \text{years to maturity date} \]

\[ P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n} \]
Table 1. Yield-to-Maturity on a $1,000 10%-Coupon Bond Maturing in Ten Years

<table>
<thead>
<tr>
<th>Price of Bond ($)</th>
<th>Yield to Maturity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,200</td>
<td>7.13</td>
</tr>
<tr>
<td>1,100</td>
<td>8.48</td>
</tr>
<tr>
<td>1,000</td>
<td>10.00</td>
</tr>
<tr>
<td>900</td>
<td>11.75</td>
</tr>
<tr>
<td>800</td>
<td>13.81</td>
</tr>
</tbody>
</table>

- When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate.
- The price of a coupon bond and the yield to maturity are negatively related.
- The yield to maturity is greater than the coupon rate when the bond price is below its face value.
Consol or Perpetuity

• A bond with no maturity date that does not repay principal but pays fixed coupon payments forever

\[ P = \frac{C}{i_c} \]

\[ P_c = \text{price of the consol} \]

\[ C = \text{yearly interest payment} \]

\[ i_c = \text{yield to maturity of the consol} \]

can rewrite above equation as this : \[ i_c = \frac{C}{P_c} \]

For coupon bonds, this equation gives the current yield, an easy to calculate approximation to the yield to maturity
Discount Bond

For any one year discount bond

\[ i = \frac{F - P}{P} \]

\( F \) = Face value of the discount bond

\( P \) = current price of the discount bond

The yield to maturity equals the increase in price over the year divided by the initial price.

As with a coupon bond, the yield to maturity is negatively related to the current bond price.
The Distinction Between Interest Rates and Returns

The payments to the owner plus the change in value expressed as a fraction of the purchase price

\[ \text{RET} = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t} \]

RET = return from holding the bond from time \( t \) to time \( t + 1 \)

\( P_t = \) price of bond at time \( t \)

\( P_{t+1} = \) price of the bond at time \( t + 1 \)

\( C = \) coupon payment

\[ \frac{C}{P_t} = \text{current yield} = i_c \]

\[ \frac{P_{t+1} - P_t}{P_t} = \text{rate of capital gain} = g \]
The Distinction Between Interest Rates and Returns (cont’d)

• The return equals the yield to maturity only if the holding period equals the time to maturity.

• A rise in interest rates is associated with a fall in bond prices, resulting in a capital loss if time to maturity is longer than the holding period.

• The more distant a bond’s maturity, the greater the size of the percentage price change associated with an interest-rate change.
The Distinction Between Interest Rates and Returns (cont’d)

• The more distant a bond’s maturity, the lower the rate of return the occurs as a result of an increase in the interest rate

• Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise
Table 2. One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

<table>
<thead>
<tr>
<th>(1) Years to Maturity When Bond Is Purchased</th>
<th>(2) Initial Current Yield (%)</th>
<th>(3) Initial Price ($)</th>
<th>(4) Price Next Year* ($)</th>
<th>(5) Rate of Capital Gain (%)</th>
<th>(6) Rate of Return (2 + 5) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10</td>
<td>1,000</td>
<td>503</td>
<td>−49.7</td>
<td>−39.7</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>1,000</td>
<td>516</td>
<td>−48.4</td>
<td>−38.4</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>1,000</td>
<td>597</td>
<td>−40.3</td>
<td>−30.3</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>1,000</td>
<td>741</td>
<td>−25.9</td>
<td>−15.9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1,000</td>
<td>917</td>
<td>−8.3</td>
<td>+1.7</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>1,000</td>
<td>1,000</td>
<td>0.0</td>
<td>+10.0</td>
</tr>
</tbody>
</table>

*Calculated with a financial calculator using Equation 3.
Interest-Rate Risk

• Prices and returns for long-term bonds are more volatile than those for shorter-term bonds

• There is no interest-rate risk for any bond whose time to maturity matches the holding period
The Distinction Between Real and Nominal Interest Rates

• **Nominal interest rate** makes no allowance for inflation

• **Real interest rate** is adjusted for changes in price level so it more accurately reflects the cost of borrowing

• Ex ante real interest rate is adjusted for expected changes in the price level

• Ex post real interest rate is adjusted for actual changes in the price level
Fisher Equation

\[ i = i_r + \pi^e \]

\( i \) = nominal interest rate
\( i_r \) = real interest rate
\( \pi^e \) = expected inflation rate

When the real interest rate is low, there are greater incentives to borrow and fewer incentives to lend. The real interest rate is a better indicator of the incentives to borrow and lend.
Figure 1. Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2011