The Interaction of Monetary Policy and Stock Returns

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Abstract

The "irrational exuberance" of the stock market in the late 1990’s lead to a discussion of the appropriate policy response by monetary authorities. Any response would be contingent on the stock market reaction to policy shocks. In this study I employ a structural VAR to estimate the response of the stock market returns to innovations in the federal funds rate. The effect of the stock market on Federal Reserve policy changes can also be examined from the empirical methodology.
1 Introduction

The effectiveness of monetary policy depends on its ability to alter the behavior of economic agents. Such policy is ultimately directed at important macroeconomic variables like real gross domestic product (GDP), consumption, investment, etc. But it is recognized that the tools available to the Federal Reserve have direct effects on interest rates and quantities of money and will only impact the ultimate policy objectives in as much as changes in interest rates and money supplies alter economic behavior. One sector that policy is believed to be able to influence is the financial sector and specifically the equity markets. Whether by altering discount rates or by influencing market participant’s expectations of future economic activity, monetary policy appears to play an important role in determining equity returns.

On the other hand there is some anecdotal evidence the behavior of equity markets also influences policy maker’s decisions. For example, many believe that the monetary tightening that began in 1999 was a direct result of the ”irrational exuberance” of equity markets over the previous three years. Whether it is appropriate or not, it is believed that the Federal Reserve implicitly targets equity returns in its policy decisions. Formal statistical evidence of the importance that equity returns play in the formulation of the Federal Reserve’s monetary policy rule has been found recently by Rigobon and Sack (2003).

An analysis of the effects of monetary policy on equity returns is therefore complicated by the endogeneity of policy decisions and the market’s reaction to those decisions. Several recent studies have used various techniques to overcome this problem. Thorbecke (1997) uses innovations in monthly federal funds rate and non-borrowed reserves to capture the effects of monetary shocks on industry level returns over the period 1967 to 1990. Unfortunately data sampled monthly are likely to miss important aspects of the relationship between stock returns and monetary policy.\(^1\) It is also the case that Thorbecke’s identification strategy is basically atheoretical and thus subject to the criticisms leveled on traditional vector autoregression (VAR) analyses by, inter alia, Cooley and LeRoy (1985).

\(^1\)It should be pointed out, however, that Thorbecke’s use of a VAR that includes industrial production, inflation and a commodity price index is more likely to capture important relationships between these variables and the policy shocks that are not possible in an analysis of daily data. The importance of the trade-off between using more variables versus using more frequently sampled data would make an interesting analysis in itself.
In a more recent study, Rigobon and Sack (2002) use daily data on stock returns and federal funds interest rates to measure the effect of monetary policy shocks on equity returns, taking advantage of the conditional volatility in asset returns to identify the policy shocks. Rigobon and Sack (2002) conjecture that the variance of the policy shocks increases on days in which the Federal Open Market Committee (FOMC) is meeting, or other similar policy-related events. They are then able to identify the policy innovations by associating these innovations with periods of marked increases in the conditional heteroscedasticity of the federal funds rate. Their results are consistent with basic intuition, that monetary tightening results in declines in equity returns. Unfortunately this identification procedure allows Rigobon and Sack to use only about 10% of the total observations available.\(^2\) A related study by D’Amico and Farka (2003) identifies the policy innovations using high frequency futures data. Monetary policy innovations are identified by changes in federal funds futures on FOMC meeting dates. D’Amico and Farka’s study reaches conclusions similar to those of Rigobon and Sack (2002) but is subject to the same data sampling criticism.

This paper is an assessment of the dynamic relationship that exists between monetary policy and equity returns. The relationship of equity prices to monetary policy is complicated by the distinction between those actions that the market has already anticipated and those it has not. In a market characterized by rational agents, only the unanticipated policy actions will effect returns since the anticipated policy will have already been imbedded in the current equity prices. An appropriate estimation procedure is the VAR framework. This technique allows the innovations in the policy variable to be interpreted as the unanticipated policy shocks. However, the VAR is a reduced form dynamic simultaneous equations system, so that endogeneity and simultaneity issues are not eliminated, and it remains necessary to impose restrictions on the empirical model in order to identify the policy innovation from changes in the policy variable caused by factors affecting the policy rule.

In this study I achieve a structural identification of the VAR by exploiting the time series properties of the underlying data. Specifically, the procedure advocated by Blanchard and Quah (1989) is extended in the direction suggested by Crowder (1995). This identification procedure has the advantage

\(^2\)Specifically, the model used by Rigobon and Sack (2002) is under-identified so that they are unable to recover any of the other structural relationships in their model. Since monetary policy shocks are identified as events and such events only occur in roughly 5% of the sample, their paired event/non-event sample only uses 10% of the available data.
that restrictions imposed to achieve an identified model, i.e. cointegration and weak exogeneity, are testable and not simply ad hoc. I control for the effects of inflation on Federal Reserve policy changes by including a commodity price index in the analysis. The results are both intuitive and significant. I find that in the appropriately specified bivariate system, policy shocks lead to immediate and opposite movements in equity returns. In contrast, exogenous equity return innovations have no immediate effect on the policy variable but eventually lead to a change in federal funds in the same direction. To control for those policy innovations associated with the Fed’s attempts to stabilize the price level, a commodity price index is added to the bivariate VAR. The results from this three variable model are consistent with those from the the model that includes only the federal funds rate and the stock returns. Furthermore, innovations in the commodity price index generate movements in the policy variable in the same direction as would be expected, i.e. increases in the price index lead to higher federal funds rates and lower equity returns.

The rest of the paper is organized as follows: section 2 describes a simple present value model of stock returns that relates two channels through which monetary policy affects equity returns. Section 3 discusses some important issues with the VAR identification methodology. Section 4 presents the empirical results and section 5 concludes. A technical appendix contains discussions of some of the more esoteric econometric issues.

2 A Simple Model of Stock Returns

The most widely used model to describe the valuation of assets is the present value model. The present value model models the stock price \( P_{t+1} \) as the present discounted value of the future expected cash flows \( D_{t+j} \). This is expressed as,

\[
P_{t+1} = E_t \left[ \sum_{j=1}^{K} \left( \frac{1}{1 + R_t} \right)^j D_{t+j} \right] + E_t \left[ \left( \frac{1}{1 + R_t} \right)^K P_{t+K} \right]
\]

where \( E_t \) is the conditional expectations operator based on information available to market participants at time \( t \), \( R_t \) is the rate of return used by market participants to discount future values and \( K \) is the investor’s time horizon or holding period. Imposing the standard transversality condition that as \( K \) gets large the last term on the right vanishes\(^3\) and dividing both sides by \( P_t \)

\(^3\)Price bubbles are the result of violations of this transversality condition.
yields an expression for stock returns.

From (1) it can be seen that monetary policy can effect stock returns in two distinct ways. First, policy can alter expected future cash flows of the firm and thereby alter the return on the firm’s stock. This channel generally relies on the effects of monetary policy on the aggregate economy. A monetary easing, a decrease in the federal funds rate, will increase the overall level of economic activity. This will in turn raise the earnings of firms in the economy and cause stock prices to rise. The effects of a monetary tightening will reduce overall expected firm profitability and stock returns.

The second, and probably more direct, way that policy effects stock returns is by altering the discount rate used by market participants. It is believed that discount factors used by equity market participants are tied in a general way to market rates of interest. Therefore, the Fed’s ability to alter discount rates is linked to its ability to alter other market rates of interest. Fuhrer (1995) finds that the federal funds rate is the source of changes in many longer term interest rates. A tighter monetary policy raises the federal funds rate, which increases the discount rate, in turn causing stock prices to decline.

Both of these channels are generally reinforcing since a tighter monetary policy usually implies both higher discount rates and lower future cash flows. In the present study, therefore, I make no attempt to distinguish between the two channels of transmission.\(^4\)

### 3 Identifying a Structural VAR

Assume that the effective federal funds rate, \(i_t\), is difference stationary while the S&P 500 return, \(r_t\), is stationary in levels. Define the stationary vector time series \(Z_t = [\Delta i_t, r_t]^\prime\). I assume throughout that \(Z_t\) has a structural moving average representation (MAR):\(^5\)

\[
Z_t = A(L)\varepsilon_t, \quad E[\varepsilon_t\varepsilon_t'] = I
\]

(2)

where \(L\) is the lag operator, \(A(L)\) is a matrix polynomial in the lag operator such that \(A(\lambda) = \sum_{j=0}^{\infty} A_j \lambda^j\) and \(\varepsilon_t = [\varepsilon_{1,t}, \varepsilon_{2,t}]\). The dynamic relationship

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\(^4\)Which of the channels is operative is of course an interesting question in and of itself, but is not addressed directly here. A recent study of these issues can be found in Bernanke and Kuttner (2003).

\(^5\)In the following discussion the deterministic components have been suppressed for ease of exposition.
between the variables in $Z_t$ can be analyzed via the impulse response functions (IRF). The IRF is defined as the $\frac{\partial Z_{i,t+k}}{\partial \varepsilon_{j,t}} = A_k(i,j)$ for $i, j = 1, 2$. An unit impulse in $\varepsilon_{j,t}$ will generate an $A_k(i,j)$ unit response in $Z_{j,t}$ $k$-periods in the future. Unfortunately the structural model is generally not estimable directly.

The Wold representation theorem implies a reduced form MAR for $Z_t$:

$$ Z_t = C(L)u_t, \quad E[u_t u'_t] = \Omega $$

where the relationship between the reduced form and structural parameters is given by $u_t = A_0 \varepsilon_t, A_j = C_j A_0$. Estimation of (3) is achieved by inverting $C(L)$ to obtain the VAR representation to which ordinary least squares (OLS) can be applied to each equation.\(^6\)

Identification of the structural model from the reduced form estimates can be achieved through suitable restrictions on the $A_0$ matrix.\(^7\) Since the reduced form covariance matrix, $\Omega$, has three distinct elements, three restrictions are already imposed on the four parameters in $A_0$ by noting that the above assumptions imply $A_0 A'_0 = \Omega$. In order to just identify the structural model, one extra restriction is needed. This extra identifying restriction often takes the form of a zero restriction on one of the off-diagonal elements of $A_0$. For example, if one assumes that monetary policy responds to stock return innovations with a one-period lag, then $A_0$ will have the form,

$$ A_0 = \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix}. $$

The restriction embodied in (4) is operationalized by taking the Cholesky decomposition of $\Omega$ such that $\Omega = PP'$ and setting $A_0 = P$. This type of restriction implies a specific contemporaneous relationship among the endogenous variables that is often incompatible with economic theory. Cooley and LeRoy (1985) have criticized such atheoretical orthogonalizations on the grounds that alternative orderings of the VAR deliver very different inferences regarding the dynamic relationships among the variables. Essentially the

\(^6\)OLS is efficient since a common number of lags are used for each equation in the VAR. If different lags are allowed in each equation then seemingly unrelated regression (SUR) will be necessary for efficient estimation.

\(^7\)It is also possible to achieve identification through other types of restrictions such as cross-equation or covariance restrictions. But imposing exclusion restrictions on $A_0$ is the most common technique in the structural VAR literature.
identification restrictions imposed in this manner are ad hoc. They suggest using economic theory to guide the choice of suitable identifying restrictions.\footnote{Bernanke (1986) and Sims (1986) independently suggested using contemporaneous restrictions that were determined by plausible economic restrictions and not necessarily recursive in structure as in the Cholesky decomposition method. However, the distinction between the Cholesky or recursive decomposition and the Bernanke-Sims decomposition is only meaningful in VARs with more than three variables. This is true since any two or three variable Bernanke-Sims just-identified VAR can be equivalently rewritten as a recursive decomposition.}

An alternative, first proposed by Blanchard and Quah (1989), that has gained some acceptance is to impose long-run neutrality type restrictions that are more firmly grounded in economic theory than contemporaneous exclusion restrictions. These restrictions represent zero restrictions on the long-run impact matrix in (2). For example, one may believe that innovations to equity returns will not permanently alter Federal Reserve policy. Such a restriction will have the form,

\begin{equation}
    A(1) = \sum_{j=0}^{\infty} A_j = \begin{bmatrix}
    a(1)_{11} & 0 \\
    a(1)_{21} & a(1)_{22}
    \end{bmatrix}.
\end{equation}

Crowder (1995) was the first to demonstrate the general equivalence between the bivariate VAR used by Blanchard and Quah (1989) and the common trends representation (CTR) model of Stock and Watson (1988). The importance of this result is that cointegration implies restrictions on the joint behavior of the variables that aid in identification of the structural system and allows the econometrician to test some of the identifying restrictions used to recover the structural model parameters from the reduced form estimates. Ribba (1997) and Fisher and Huh (1999) extend Crowder’s result by demonstrating that in a cointegrated system that is characterized by weak exogeneity, the contemporaneous identification is valid as long as the number of weakly exogenous variables is equal to the number of common trends. Furthermore, since both cointegration and weak exogeneity are testable, it may be possible to completely identify a structural model using testable restrictions making them less susceptible to the criticisms of Cooley and LeRoy (1985).\footnote{The technical appendix provides a detailed explanation of the preceding claims.}

In the empirical analysis that follows, I test each of the variables for a unit root, using both univariate and multivariate procedures. I conclude that there is one cointegrating relationship between stock returns and the federal
funds rate. I then estimate structural VAR models under the restrictions
given in (4) and (5) and compare the results. The IRFs using restriction
matrix (4) generate results that are consistent with the present value model
in Section 2. The IRFs generated using restriction matrix (5) yield counter-
intuitive results. I then find that, for relevant lag truncations in the VAR,
the federal funds rate is weakly exogenous in the bivariate model of stock
returns and the federal funds rate. This result implies that the restriction
matrix in (4) is a valid identifying tool. Adding a commodity price index
variable to the model to control for the effects of inflation on the monetary
policy rule does not change the basic results.

4 The Empirical Analysis

The data used in the analysis are the effective federal funds rate obtained
from the Federal Reserve Bank of St. Louis and the S&P 500 return ob-
tained from EconStats. In addition to the bivariate analysis, I control for
the effects of inflation on the monetary policy innovation by including the
Commodity Research Bureau’s (CRB) commodity spot index. The sample
is daily from February 3, 1970 to June 16, 2003. The stock returns are cal-
culated as the annualized log change in the daily closing price of the S&P
500 index. The data are plotted in Figure 1.

For the data to be consistent with the identification procedure outlined
above, the federal funds rate should be difference stationary or I(1) and the
stock return should be level stationary or I(0). Whether the CRB index is I(0)
or I(1) will determine whether the three-variable system can be characterized
as having one cointegrating vector or two. I employ two methodologies to
test these hypotheses. The first are univariate unit root testing procedures,
including a recently proposed test by Ng and Perron (2001). The second is
the multivariate unit root test proposed by Johansen (1991).

10 The web links for the data sets are http://research.stlouisfed.org/fred2/ and
http://www.econstats.com/, respectively.
11 The anonymous referee deserves credit for suggesting the inclusion of this vari-
  able in the analysis. Information on this index can be found at the CRB’s web site
  http://www.crbtrader.com/crbindex/.
12 The technical appendix provides a detailed description of each estimator and the
  relevant statistical tests.
4.1 Univariate Analysis

4.1.1 Dickey-Fuller Tests

The most commonly used unit root test in the literature is the augmented Dickey-Fuller test (ADF τ-test) of Said and Dickey (1984). This test is based on the regression,

\[ \Delta y_t = \mu + \rho y_{t-1} + \sum_{j=1}^{k} \gamma_j \Delta y_{t-j} + \nu_t \]  \hspace{1cm} (6)

where \( k \) is chosen to eliminate residual autocorrelation. The null hypothesis of a unit root is tested by comparing the \( t \)-statistic on \( \rho \) to an appropriate critical value. The distribution of this test is non-standard. Critical values have been tabulated by MacKinnon (1996).

ADF unit root test results are often sensitive to the number of lagged terms included in the regression. Figure 2 provides some evidence on the robustness of the unit root inference for each series. This figure displays the augmented Dickey-Fuller (ADF) test statistics calculated by varying the number of lags from 1 to 100. The statistics have all been divided by the 5% critical value so that values of the test greater than one imply statistical significance. For the federal funds rate series, the unit root null hypothesis is rejected only at very small lags in the ADF regression. On the other hand, the S&P 500 return series yields a rejection of the unit root null at all lag lengths. Finally, the CRB index yields no rejections of the unit root null at any lag length considered.\(^{13}\)

4.1.2 Optimal Unit Root Tests

It is well known that the ADF tests have low power against local stationary alternatives. Elliot, Rothenberg and Stock (1996) (ERS) develop a feasible point optimal test that relies on local generalized least squares (GLS) detrending to increase the power of the unit root tests. Their test is denoted \( DF^{GLS} \). A second serious problem associated with unit root testing is that the standard tests may suffer from serious size distortions when the data generating process (DGP) has negative moving average (MA) terms.\(^{14}\) Perron

\(^{13}\)It is the case, however, that the ADF tests on the CRB index would reject the unit root null at the 10% level for almost all lag lengths considered.

and Ng (1996) extend the work of ERS by developing modified versions of the Phillips and Perron (1988) tests that have much better size properties than the conventional tests but also retain the power of the $DF^{GLS}$. These unit root tests are based on the local GLS detrending method and, in addition, use an autoregressive spectral density estimator of the long-run variance. The two tests are labelled the $MZ_\rho$ and the $MZ_\tau$ tests. Ng and Perron (2001) suggest that these two tests have similar power to the $DF^{GLS}$ test of Elliot, Rothenberg and Stock (1996) but also have superior size properties in the presence of MA disturbances. The decrease in size and increase in power are enhanced when one chooses the lag length based on the modified information criteria (MIC) developed in Ng and Perron (2001).

Employing the GLS detrending procedure of ERS and the modified information criterion of Ng and Perron (2001) to choose the appropriate lag truncation of 68 yields a $DF^{GLS}$ statistic for the federal funds rate of $-2.86$ which, when compared to the 5% critical value of $-2.91$, cannot reject the unit root null. The same statistic calculated for the return series with an optimally chosen 35 lags yields a value of $-5.98$ which does lead to a rejection of the unit root null. Similarly, the $MZ_\rho$ test of Ng and Perron for the federal funds series is $-17.13$ which is not statistically significant at the 5% level, while the same test for the return series is $-19.82$, which is statistically significant at the 5% level and thus rejects the unit root null. The same tests applied to the CRB index result in a $DF^{GLS}$ test statistic of $-1.07$ using an optimally chosen lag of one and a $MZ_\rho$ statistic of $-2.52$, neither of which is statistically significant at conventional levels. Therefore the evidence suggests that the federal funds rate and the CRB index are I(1) series while the S&P 500 return series is I(0).

unit root tests approaches unity as the sum of the MA parameters in the univariate process approaches negative one. The relevance for the series examined in this study can be demonstrated by noting that when each series is fit to an ARMA(1,3) model the sum of the MA parameters is $-0.62$, $0.04$ and $-0.72$ for the federal funds rate, commodity price index and the equity returns series, respectively.

$^{15}$Phillips and Perron (1988) correct for serial correlation using a semi-parametric correction to the test statistics that is based on an estimate of the long-run variance at frequency zero.
4.2 The Relationship Between Returns and Monetary Policy

In this section, I examine the multivariate relationship between stock returns and monetary policy by using the VAR methodology. Following the earlier discussion, I first test for the number of unit roots in the VAR using the Johansen procedure. Finding that the VAR has a reduced number of common trends allows the VAR to be written as an error-correction mechanism or VECM. From the VECM I am able to conduct dynamic response analysis to determine the temporal relationship between equity returns and monetary policy.

4.2.1 Johansen Tests

The Johansen procedure is based upon the vector error correction model (VECM) in equation (7),

\[
\Delta X_t = \mu + \alpha \beta' X_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta X_{t-j} + \varepsilon_t \tag{7}
\]

where \( X_t \) is a \( p \times 1 \) vector of at most I(1) series, \( \beta \) is a \( p \times r \) matrix whose \( r \) columns represent the cointegrating vectors among the variables in \( X_t \), and \( \alpha \) is a \( p \times r \) matrix whose \( p \) rows represent the error correction coefficients. The Johansen test for cointegration, called the trace test, tests the rank of the \( p \times p \) product matrix \( \alpha \beta' \) such that reduced rank, rank less than \( p \), implies cointegration.\(^{16}\)

Like their univariate counterparts, the Johansen tests can be sensitive to lag specification in the VAR. Figure 3 plots the trace test statistics for the hypothesis of zero stationary relations (or no cointegration vectors) and the hypothesis of one stationary relation in the bivariate system including the federal funds rate and the stock return series. These statistics have also been normalized by their appropriate 5% critical values so that values greater than one imply rejection of the null hypothesis.\(^{17}\) The top panel represents the tests of the null of zero stationary relations or that both series in the VAR are non-stationary and not cointegrated. This hypothesis is

\(^{16}\) The technical appendix provides a more detailed description of the estimator and tests of restrictions on \( \beta \).

\(^{17}\) The critical values were taken from MacKinnon, et al. (1996).
rejected at all lag truncations considered. The bottom panel displays the test statistics under the null that only one stationary relation exists in the VAR versus the alternative that both series are stationary. Except at very short lag truncation values, this hypothesis is not rejected. This supports the conclusions drawn from the univariate tests in which the federal funds rate is found to be I(1) and the stock return is I(0).

Figure 4 displays the likelihood ratio tests of the null hypothesis that each series can be excluded from the stationary or cointegrated relationship. These represent tests of the stationarity of a single series conditional on one cointegration vector in the VAR. Again, these statistics are calculated for all lag choices in the VAR from 1 to 100 and are normalized by 3.84, the 5% critical value in the $\chi^2(1)$ distribution so that values exceeding unity are statistically significant. The top panel displays the tests that stock returns can be excluded from the estimated stationary relation, or, equivalently, that the federal funds rate is stationary. This hypothesis is rejected at all lag choices, implying that the federal funds rate is not stationary by itself. The bottom panel of Figure 4 displays the analogous test for the return series. In this case the null hypothesis is never rejected. Thus the S&P 500 return series is the stationary variable in the two-variable VAR.

4.2.2 Lower Triangular Contemporaneous Identification

Figure 5 displays the impulse response functions (IRFs) for the federal funds rate and S&P 500 return obtained using identification restrictions implied by (4). Under this restriction the federal funds rate responds to innovations in the equity return with a one-period lag. Included in the figure is the 90% confidence interval about each IRF.\footnote{These were calculated using the Monte Carlo procedure suggested by Sims and Zha (1999).} The pattern of the responses coincides considerably with basic intuition. A positive federal funds rate innovation permanently raises the federal funds rate by about fifteen basis points in the long run and reduces equity returns by about 1.2% initially. On the other hand a positive S&P 500 innovation induces no statistically significant policy response for about 40 days. At this time the federal funds rate increases. This supports the idea that the Federal Reserve responds to equity markets when formulating policy.
4.2.3 Blanchard-Quah Identification

From the unit root tests it is clear that the bivariate VAR of federal funds rate and S&P 500 return are similar to the VAR specification used by Blanchard and Quah (1989). By imposing the restriction that innovations in the S&P 500 return have no long-run impact on the federal funds rate, the model is just identified and can be given a structural interpretation. Figure 6 displays the impulse response functions (IRFs) under this identification scheme. The responses under this identification structure are counterintuitive. Specifically an unanticipated increase in the federal funds rate leads to an increase in equity returns. Similarly, an unanticipated increase in equity returns leads to a (temporary by construction) fall in the federal funds rate. This seems at odds with conventional wisdom and the theoretical model presented in section 2.

4.2.4 Weak Exogeneity

From the tests results presented earlier it is clear that the bivariate VAR under analysis meets the conditions laid out in section 3 for treating the system as cointegrated. As discussed in section 3 and detailed in the appendix, if one of the variables is weakly exogenous, then the $A_0$ matrix given in equation (4), i.e. imposing a contemporaneous exclusion restriction, will satisfy the identification conditions of the structural model.

Weak exogeneity can be tested from equation (7) by simply testing for the statistical significance of $\alpha$ in each equation of the VECM. Since all of the variables in (7) are stationary, conventional $t$-tests will be appropriate. Figure 7 displays tests for the weak exogeneity of each variable in the VAR for all lag choices from 1 to 100. It is interesting that the S&P 500 return series is never found to be weakly exogenous but for most lag truncation choices in the VAR, the federal funds rate is weakly exogenous. Specifically, the federal funds rate is found to weakly exogenous for VAR lag lengths of 1 to 48 and 70 to 100.

This begs the question of what is the most appropriate lag truncation for the federal funds rate series in the VAR? I use eight different lag selection techniques to answer this question with six different answers, presented in Table 1. Most importantly, none of the suggested lag lengths in table 1 lie in the range in which the federal funds rate is endogenous.

Figure 8 displays the IRFs assuming the weak exogeneity of the federal
funds rate which leads to a lower triangular $A_0$ matrix. The IRFs under this identification strategy look more like those obtained under the contemporaneous restriction procedure than those from the BQ procedure. The dynamic behavior of the variables conforms to standard intuition. A positive innovation in the federal funds rate leads to a negative reaction by equity returns. The magnitudes of the equity return response are smaller under this identification scheme. A 100 basis point positive innovation to the federal funds rate results in an approximate 0.3% decline in equity returns in the first period. And a positive shock to equity returns ultimately leads to an increase in the federal funds rate, but it is not always statistically significant.

The impulse responses from a contemporaneous identification restriction were more consistent with economic theory than those obtained by imposing long-run neutrality restrictions. Specifically, shocks to the federal funds rate, which are interpreted as policy innovations, result in responses to equity returns in the opposite direction, while shocks to equity returns result in movements in the policy variable, federal funds interest rates, in the same direction. The weak exogeneity of the federal funds rate validates the use of a contemporaneous restriction to identify the structural innovations.

4.3 Controlling for Inflation in the Monetary Policy Rule

It is generally believed that one of the most important variables used by the Fed to set monetary policy is general price inflation. It may even be the case that the Fed has adopted a specific inflation target. The possibility exists that the innovations identified in the bivariate model are in fact endogenous responses to the effects of price inflation rather than exogenous policy shocks. To investigate this possibility, I add the CRB index to the analysis and examine how the results differ.

4.3.1 Johansen Tests

Figure 9 plots the trace test statistics for the hypotheses of zero stationary relations (or no cointegration vectors), one stationary relation and two stationary or cointegrating relations in the three-variable system including the federal funds rate, the CRB index and the stock return series. Again, these statistics have been normalized by their appropriate 5% critical values so that values greater than one imply rejection of the null hypothesis. The top panel
represents the tests of the null of zero stationary relations or that all three series in the VAR are non-stationary and not cointegrated. This hypothesis is rejected at all lag truncations considered. The middle panel displays the trace tests of the hypothesis that there is no more than one cointegrating relation among the three variables. This hypothesis is only rejected at low lag lengths. The bottom panel displays the test statistics under the null that all three series are stationary. The results are consistent with the univariate tests where both the federal funds rate and the CRB index are I(1) and the S&P 500 return is I(0).

Figure 10 displays the likelihood ratio tests of the null hypothesis that two of the series can be excluded from the one stationary relationship. These tests represent tests of the stationarity of a single series conditional on one cointegration vector in the VAR. Again these statistics are calculated for all lag choices in the VAR from 1 to 100 and are normalized by 5.99, the 5% critical value in the $\chi^2(2)$ distribution. The top panel displays the tests that the federal funds rate is stationary. This hypothesis is rejected at all lag choices, implying that the federal funds rate is not stationary by itself. The middle panel of Figure 10 displays the analogous test for the return series. In this case the null hypothesis is never rejected implying that the S&P 500 return series is stationary. The bottom panel tests whether the CRB index is stationary. This hypothesis is also rejected at all lags considered. These results are consistent with those of the univariate tests and with the specification of the VAR as a cointegrated system.

4.3.2 Weak Exogeneity

Figure 11 displays tests for the weak exogeneity of each variable in the trivariate VAR for all lag choices from 1 to 100. The S&P 500 return series is never found to be weakly exogenous but for most lag truncation choices in the VAR, the federal funds rate and the CRB index are weakly exogenous.

Figure 12 displays the IRFs under the restriction that the federal funds rate and the CRB index are both weakly exogenous. The IRFs under these identification restrictions are also consistent with a priori expectations. A positive innovation in the federal funds rate leads to a negative reaction by equity returns and the CRB index. An innovation in the CRB index has no

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19 These tests are operationalized by excluding two of the variables from the cointegrating vector. A rejection then implies that at least one of the excluded variables belongs in the stationary or cointegrating relation.
immediate statistically significant affect on either the federal funds rate or stock returns, but eventually leads to a monetary tightening and a decline in equity returns. Finally, the innovation in stock returns has no immediate affect on the federal funds rate, but consistent with the idea that the Federal Reserve policy uses information about equity returns, a positive return shock leads to an eventual increase in the federal funds rate and also a decline in the CRB index.

5 Conclusions

This study analyzes the dynamic relationship that exists between equity returns and monetary policy by employing structural VAR analysis. I compare the results from three different identification procedures and find that those associated with a standard Cholesky decomposition and those generated by imposing weak exogeneity on the federal funds rate lead to the most plausible results. Using the Blachard-Quah identification strategy yields responses that are inconsistent with theory and of implausibly large magnitudes. Including the CRB commodity price index to control for the effects of inflation on Federal Reserve policy does not overturn the results.
A Technical Appendix

A.1 Structural VAR identification

Because economic theory generally has more specific implications for long-run behavior, Blanchard and Quah (1989) suggested imposing a long-run neutrality restriction on the VAR system. The form of this restriction in the current analysis implies that shocks to stock returns have no long-run effect on the federal funds rate. This means that the Fed will only respond in a temporary way to innovations in the equities markets. The total impact matrix from (2) will be lower triangular:

\[
A(1) = \sum_{j=0}^{\infty} A_j = \begin{bmatrix}
A_{11}(1) & 0 \\
A_{21}(1) & A_{22}(1)
\end{bmatrix}.
\] (8)

From the relationship between the structural and reduced form VAR models, i.e. \( A(1) = C(1)A_0 \), the identification restriction in (8) can be imposed by suitable specification of \( A_0 \).

Consider an equivalent representation of the system given in (3) by defining \( X_t = [i_t, r_t]' \). \( X_t \) is a cointegrated system under the assumptions made earlier. Specifically, \( X_t \) is an I(1) system, since no individual time series in \( X_t \) is integrated of an order higher than one, and there exists a linear combination that is integrated of order zero. This linear combination is given by \( \beta = [0, 1]' \).

Engle and Granger (1987) demonstrate that the cointegrated system \( X_t \) has an error-correction model (ECM) representation as,

\[
\Delta X_t = \alpha \beta' X_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta X_{t-j} + \nu_t \quad E[\nu_t \nu_t'] = \Sigma
\] (9)

where \( \alpha \) is the 2 x 1 matrix of equilibrium adjustment or error-correction coefficients. Campbell and Shiller (1988) prove that the ECM representation of (9) has an equivalent VAR representation as,

\[
B(L)Z_t = u_t \quad E[u_t u_t'] = \Omega
\] (10)

\(^{20}\)While this specification of the cointegrating relationship is unusual it does meet the definitional requirements of a cointegrated system.
where \( Z_t = [\Delta X_{1t}, \beta' X_t]' \). This is just the definition of \( Z_t \) from (3) so that (10) is the autoregressive representation of the reduced form MAR system in (3) where \( C(L)^{-1} = B(L) \).

From Granger’s Representation Theorem the moving average representation of (9) exists and can be written as,

\[
\Delta X_t = D(L)^{-1} \nu_t
\]

(11)

where the matrix polynomial \( D(\lambda) \) can be decomposed as \( D(\lambda) = D(1) + (1 - \lambda) D^*(\lambda) \), with \( D^*(\lambda) = (D(\lambda) - D(1))(1 - \lambda)^{-1} \). Johansen (1991) shows that when \( X_t \) is integrated of order one or less then the matrix \( \xi = (\alpha' \Gamma(1) \beta_\perp)^{-1} \), \( \Gamma(1) = I - \sum_{j=1}^{K-1} \Gamma_j \), exists and is nonsingular. The total impact matrix from (11) has the form \( D(1) = \beta_\perp \xi \alpha' \) where \( \beta_\perp \) and \( \alpha_\perp \) are orthogonal complements to \( \beta \) and \( \alpha \) such that \( \beta' \beta_\perp = 0 \) and \( \alpha' \alpha_\perp = 0 \). Using these relations allows (11) to be written in common trends representation (CTR) form as,

\[
X_t = D(1)(1 - L)^{-1} \nu_t + D^*(L) \nu_t
\]

(12)

where \( (1 - L)^{-1} \nu_t = \sum_{s=0}^{t} \nu_s = \tau_t \) are the common (random walk) trend components in \( X_t \). The total impact matrix \( D(1) \) will have reduced rank of one allowing the interpretation of (12) as a common trends representation.

Given the earlier assumption regarding \( X_t \), the orthogonal complement to \( \beta \) can be given by \( \beta_\perp = [1, 0]' \). Thus the CTR can be explicitly written as,

\[
X_t = \begin{bmatrix} i_t \\ r_t \end{bmatrix} = \begin{bmatrix} \xi_1 \alpha_{1\perp} & \xi_2 \alpha_{2\perp} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tau_{1t} \\ \tau_{2t} \end{bmatrix} + \begin{bmatrix} C_{11}^*(L) & C_{12}^*(L) \\ C_{21}^*(L) & C_{22}^*(L) \end{bmatrix} \begin{bmatrix} \nu_{1t} \\ \nu_{2t} \end{bmatrix}.
\]

(13)

From the CTR in (13) it can be seen that the common trend is proportional to \( \alpha'_t \nu_t \). Given that the structural model in (2) implies that the transitory innovations are orthogonal to the permanent innovations, it must be the case that the transitory innovations are proportional to \( \alpha' \Sigma^{-1} \nu_t \). In order to recover the structural innovations (and other structural parameters) Hansson (1999) demonstrates a suitable choice for \( A_0^{-1} \) is given by,

\[
A_0^{-1} = \begin{bmatrix} (\alpha_1' \Sigma \alpha_\perp)^{-1/2} \alpha_\perp & (\alpha_2' \Sigma \alpha_\perp)^{-1/2} \alpha_\perp \\ (\alpha_1' \Sigma^{-1} \alpha)^{-1/2} \alpha \Sigma^{-1} & (\alpha_2' \Sigma^{-1} \alpha)^{-1/2} \alpha \Sigma^{-1} \end{bmatrix}
\]

(14)

\(^{21}\)In general the specification of the cointegrated system in (10) will be algebraically different from that in (3). But because of the special nature of the cointegrating vector in this analysis, i.e. \( \beta = [0, 1]' \), the two representations are algebraically the same. Thus \( \nu_t = u_t \) and \( \Omega = \Sigma \).
as this ensures that the permanent and transitory innovations are uncorrelated and delivers a lower triangular $A(1)$ matrix as in Blanchard and Quah (1989).\footnote{From (2), (3) and (10) it is seen that $A(1) = C(1)A_0 = B(1)^{-1}A_0 = (A_0^{-1}B(1))^{-1}$. Given that $B(1) = \begin{bmatrix} B_{11}(1) & \alpha_1 \\ B_{21}(1) & \alpha_2 \end{bmatrix}$ it will be the case that $A(1)$ is lower triangular.}

But another interesting result arises from (14). If the federal funds rate is weakly exogenous with respect to the long-run parameters, i.e. $\alpha$ and $\beta$, then $\alpha_{2\perp} = 0$ and any $A_0$ matrix would deliver a lower triangular $A(1)$ matrix since $A_0$ itself will be lower triangular. Under the weak exogeneity restriction, specifying $A_0$ as in (4) is warranted since it is consistent with the underlying data generating process of the system. This result can be attributed to Ribba (1997) and has been generalized by Fisher and Huh (1999) to case of more than two variables. The key requirement is that the number of weakly exogenous variables is equal to the number of common trends. In the three-variable model that includes the CRB index there are two common trends and both the federal funds rate and the CRB index are weakly exogenous satisfying the conditions laid out above. What makes this result so appealing is that it can be tested using conventional hypotheses on the parameters of the $\alpha$ (or $\alpha_{\perp}$) matrix.

### A.2 Univariate Unit Root Tests

The tests proposed by Ng and Perron are motivated by the DGP in (15),

$$y_t = d_t + u_t, \quad u_t = \rho u_{t-1} + v_t$$

where $v_t = \varphi(L)e_t = \sum_{j=0}^{\infty}\varphi_j e_{t-j}, d_t = \zeta'z_t = \sum_{i=0}^{p}\zeta_i t^i$ for $p = 0, 1$. ERS suggest using a GLS detrending method to improve the power of unit root tests. For any series $\{x_t\}_{t=0}^{T}$ define $(x_0^{\alpha}, x_t^{\alpha}) \equiv (x_0, (1-\alpha L)x_t)$ for some chosen $\alpha = 1 + c/T$. The GLS detrended series is defined as,

$$\tilde{y}_t \equiv y_t - \hat{\zeta}'z_t$$

where $\hat{\zeta}$ minimizes $S(\hat{\pi}, \hat{\zeta}) = (y^{\hat{\pi}} - \hat{\zeta}'z^{\hat{\pi}})'(y^{\hat{\pi}} - \hat{\zeta}'z^{\hat{\pi}})$. ERS suggest choosing $\pi = -7.0$ for $p = 0$ and $\pi = -13.5$ for $p = 1$. The $DF_{GLS}$ statistic is the $t$-statistic testing the null that $\rho = 0$ in equation (17).
Ng and Perron recommend two tests that have similar power to the \(DF^{GLS}\) but that also have superior size properties in the presence of MA errors. These tests are \(MZ_{\rho}\), \(MZ_{t}\), and \(MSB\), collectively referred to as the \(M\) tests. These are defined as,

\[
MZ_{\rho} = (T^{-2} \bar{y}_{t}^{2} - s_{AR}^{2})(2T^{-2} \sum_{t=1}^{T} \bar{y}_{t-1}^{2})^{-1}
\]

and

\[
MZ_{t} = MZ_{\rho} \times MSB.
\]

All three tests are based on \(s_{AR}^{2}\), an autoregressive estimate of the spectral density at frequency zero of \(v_{t}\). This estimate is calculated as,

\[
s_{AR}^{2} = \frac{\hat{\sigma}_{k}^{2}}{[1 - \gamma(1)]^2}
\]

where \(\gamma(1) = \sum_{i=1}^{k} \gamma_{i}\) and \(\hat{\sigma}_{k}^{2} = (T - k)^{-1} \sum_{t=k+1}^{T} \hat{\epsilon}_{tk}^{2}\) and \(\gamma_{i}\) and \(\{\hat{\epsilon}_{tk}\}\) are taken from estimation of (17) using OLS. The only piece left is to specify a lag truncation parameter \(k\). Ng and Perron suggest using a modified information criteria (MIC) as in (21),

\[
MIC(k) = \ln(\hat{\sigma}_{k}^{2}) + \frac{C_{T}(\tau_{T}(k) + k)}{T - k_{\max}}
\]

where \(\tau_{T}(k) = (\hat{\sigma}_{k}^{2})^{-1} \hat{\rho} \sum_{t=k_{\max}+1}^{T} \bar{y}_{t}^{2}\) and \(k_{\max}\) is the largest lag truncation considered. Ng and Perron show that if \(C_{T} = \ln(T - k_{\max})\) then this reduces to their \(MBIC\) or modified Bayesian information criteria.

### A.3 Multivariate Unit Root Tests

The Johansen procedure is based upon the vector error correction model (VECM) in equation (7). Maximum likelihood estimation of (7) can be carried out by applying reduced rank regression. Johansen (1988, 1991) suggests first concentrating out the short-run dynamics by regressing \(\Delta X_{t}\) and \(X_{t-1}\) on \(\Delta X_{t-1}, \Delta X_{t-2}, \ldots, \Delta X_{t-k+1}\) and \(\mu\) and saving the residuals as \(R_{0t}\) and \(R_{1t}\), respectively. Calculate the product moment matrices \(S_{ij} = T^{-1} \sum_{t=1}^{T} R_{it} R_{jt}'\) and solve the eigenvalue problem \(\lambda S_{11} - S_{10} S_{01}^{-1} S_{01} = 0.\) The
likelihood ratio statistic testing the null hypothesis of at least $r$ cointegrating relationships in $X_t$ is given by $-T \sum_{i=r+1}^{p} \ln(1 - \hat{\lambda}_i)$ and is called the trace statistic by Johansen (1988, 1991). The distribution of this statistic is non-standard and depends upon nuisance parameters. Critical values have been tabulated by MacKinnon et al. (1996) using response surface regressions.

The estimate of $\beta$, $\hat{\beta}$, is given by the $r$-largest eigenvectors associated with the eigenvalues $\hat{\lambda}$. Hypothesis tests on $\hat{\beta}$ can be conducted using likelihood ratio (LR) tests with standard $\chi^2$ inference. Let the form of the linear restrictions on $\beta$ be given by $\beta = H \varphi$ where $H$ is a $p \times s$ matrix of restrictions and $\varphi$ is a $s \times r$ matrix of unknown parameters. The LR test statistic is given by,

$$T \sum_{i=1}^{r} \ln \left[ \frac{(1 - \tilde{\lambda}_i)}{(1 - \hat{\lambda}_i)} \right] \sim \chi^2_{r(p-s)}$$  \hspace{1cm} (22)

where $\tilde{\lambda}_i$ are the eigenvalues from the restricted MLE.
References


Table 1: Tests for Lag Truncation of Federal Funds Equation in VAR

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Suggested Lag</th>
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<tbody>
<tr>
<td>AIC</td>
<td>96</td>
</tr>
<tr>
<td>BIC</td>
<td>26</td>
</tr>
<tr>
<td>Modified AIC</td>
<td>35</td>
</tr>
<tr>
<td>Modified BIC</td>
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<tr>
<td>Ljung-Box Test</td>
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<td>Lagrange Multiplier Test</td>
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<tr>
<td>General-to-Specific</td>
<td>96</td>
</tr>
<tr>
<td>Specific-to-General</td>
<td>6</td>
</tr>
</tbody>
</table>

**Note:** AIC is the Akaike criterion. BIC is the Schwarz Bayesian criterion. Modified AIC and Modified BIC are criteria suggested by Ng and Perron (2001). Ljung-Box is the Q-statistic with degrees of freedom equal to 180. LM is the Lagrange multiplier test with degrees of freedom equal to 180. General-to-specific starts with the longest hypothesized lag (100) and sequentially tests the last lag coefficient for statistical significance until the hypothesis can be rejected. Specific-to-General starts with zero lags and adds lags until the last lag added is statistically insignificant.
Figure 1

S&P 500 Daily Returns
February 1970 to June 2003

Effective Federal Funds Rate
February 1970 to June 2003

CRB Spot Index
February 1970 to June 2003
Unit Root Tests on the Federal Funds Rate

Unit Root Tests on the S&P 500 Return

Unit Root Tests on the CRB Spot Index
Tests of Stationarity of the Federal Funds Rate

Tests of Stationarity of the S&P 500 Return

Figure 4
Impulse Response Functions

Response of FFUNDS

Response of RETURN

Figure 5
Impulse Response Functions

Response of FFUNDS

Response of RETURN

Figure 6
Figure 7

Tests of Weak Exogeneity of the Federal Funds Rate

Tests of Weak Exogeneity of the S&P 500 Return
Impulse Response Functions

Response of FFUNDS

Response of RETURN

Figure 8
Impulse Response Functions

Response of FFUNDS

Response of CRB

Response of RETURN

Figure 12