International Evidence on the Fisher Relation

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Abstract

This paper uses data on short-term nominal interest rates and inflation rates over the last three decades collected from eight industrialized nations to examine the whether the Fisher relation has empirical support internationally. The results indicate that the Fisher relation has a great deal of support with all eight countries exhibiting cointegration between nominal rates and inflation. In seven of the eight countries, the Fisher effect estimate is statistically greater than one implying significant tax effects as hypothesized by Darby (1975). Finally, our analysis of the error correction models reveal that inflation is weakly exogenous and therefore the source of the common trend in each system.

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1 Introduction

Recently there has been a revival in the study of the relationship between nominal interest rates and inflation. The Fisher equation is a simple way to model this relationship. Fisher (1930) described an environment where nominal rates respond one-for-one to changes in the expected decline in the purchasing power of money. When interest income is taxed this Fisher effect is greater than one as suggested by Darby (1975). In the 1970s and 1980s numerous studies produced evidence that the response of nominal interest rates to changes in inflation were less than one. Tanzi (1980) concluded that economic agents suffered from a form of irrationality he termed as "fiscal illusion." Tobin (1965, 1969) had earlier suggested that inflation was negatively correlated with real interest rates since increases in inflation caused people to shift out of nominal assets and acquire real assets. This Tobin effect offered an explanation for low Fisher effect estimates that did not rely on investor irrationality. Lucas (1980) points out that the Tobin effect is a short-run phenomenon if one assumes that money is superneutral in the long run.

Mishkin (1992) was one of the first to suggest that due to the non-stationarity of nominal interest rates and inflation a possible source of the low Fisher effect estimates is the spurious regression problem discussed by Granger and Newbold (1974). He correctly pointed out that the proper treatment of the Fisher relation is as a cointegrated system as in Engle and Granger (1987). Mishkin used the Engle-Granger OLS procedure to estimate the Fisher effect but was unable to make any strong conclusions due to the large standard errors of the estimated parameters. Subsequent studies used more efficient estimation procedures.

The focus of the recent research has been on the U.S. It is of interest whether nominal interest rates and inflation of other industrialized countries also exhibit a long-run Fisher relation. For example, a necessary condition, but not sufficient, for real interest rates to be equalized internationally is that the Fisher relation holds in each country individually.

This paper uses data on short-term nominal interest rates and inflation rates over the last three decades collected from eight industrialized nations to examine the whether the Fisher relation has empirical support internationally. The results indicate that the Fisher relation has a great deal of support with all eight countries exhibiting cointegration between nominal rates and inflation. In seven of the eight countries, the Fisher effect estimate is statistically greater than one implying significant tax effects as hypothesized by Darby (1975). Finally, our analysis of the error correction models reveal that inflation is weakly exogenous and therefore the source of the common trend in each system.

The next section outlines the Fisher relation. In section 3 the econometric methodology is discussed. The empirical results are presented in section 4 and section 5 concludes.

2 The Fisher Relation

In a standard asset pricing model where $k$-period real and nominal default-free bonds both trade, the following relation will hold between their respective continuous compound rates
of return;

\[ i^k_t = E_t \pi_{t+k} + r^k_t + \frac{1}{2} \text{Var}_t[\pi_{t+k}] - \text{Cov}_t[\pi_{t+k}, q_{t+k}] \] (1)

where \( i^k_t \) is the \( k \)-period return on the nominal bond, \( E_t \pi_{t+k} \) is the expected inflation over the life of the bond and \( q_{t+k} \) is the real stochastic discount factor.\(^3\) This form of the Fisher relation specifies that the nominal interest rate is the sum of expected inflation, the real interest rate and an inflation risk premium which depends on the second moments of the joint distribution of \( \pi \) and \( q \).

If we assume that expectations are unbiased and that the risk premium is stationary (1) may be written in regression form as in (2),

\[ i^k_t = a_0 + a_1 \pi_{t+k} + \eta_t \] (2)

where \( \eta_t \) is a possibly autocorrelated stationary residual and \( a_0 \) contains the means of the real rate plus the risk premium. The parameter of greatest interest is \( a_1 \) the measure of the Fisher effect. In Fisher’s original formulation \( a_1 = 1 \), but as demonstrated by Darby (1975), when nominal interest income is subject to taxation one should expect \( a_1 > 1 \). In the following section the Johansen (1988, 1991) method is used to test for the existence of a cointegrating relation between nominal interest and inflation rates for eight industrialized nations. Hypothesis tests on the cointegration space will reveal whether \( a_1 \) is consistent with Fisher’s original specification or Darby’s tax-adjusted specification.
3 Estimation Procedure

The estimates of the Fisher effect are obtained from Johansen’s (1988) gaussian MLE. Assume that the $p^{th}$ order vector time series $X_t$ follows a finite order vector autoregressive (VAR) process given in (3),

$$B(L)X_t = \mu_t + \varepsilon_t$$

(3)

where $B(L)$ is a $k^{th}$ order matrix polynomial in the lag operator, $\mu_t$ contains deterministic components which may depend upon time and $\varepsilon_t$ is a vector white noise error term. Equation (3) can be transformed into error-correction model (ECM) form as in (4).

$$\Delta X_t = \mu_t + \alpha \beta' X_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta X_{t-j} + \varepsilon_t$$

(4)

The $p \times r$ matrix $\beta$ in (4) represents the cointegrating vectors while $p \times r$ matrix $\alpha$ is the matrix of error-correction coefficients. Maximum likelihood estimation of (4) can be carried out by applying reduced rank regression. Johansen (1988) suggests first concentrating out the short-run dynamics by regressing $\Delta X_t$ and $X_{t-1}$ on $\Delta X_{t-1}$, $\Delta X_{t-2}$, ..., $\Delta X_{t-k+1}$ and $\mu_t$ and saving the residuals as $R_{0t}$ and $R_{1t}$, respectively. Calculate the product moment matrices $S_{ij} = T^{-1} \sum_{t=1}^{T} R_{it} R_{jt}'$ and solve the eigenvalue problem $|\lambda S_{11} - S_{10} S_{00}^{-1} S_{01}| = 0$. The likelihood ratio statistic testing the null hypothesis of at least $r$ cointegrating relationships in $X_t$ is given by $-T \sum_{i=r+1}^{p} \ln(1 - \hat{\lambda}_i)$ and is called the trace statistic by Johansen (1988). The distribution of this statistic is non-standard and depends upon nuisance parameters. Critical
values have been tabulated by MacKinnon et al. (1996) using response surface regressions.

The estimate of $\beta$, $\hat{\beta}$, is given by the $r$-largest eigenvectors associated with the eigenvalues $\hat{\lambda}$. Hypothesis tests on $\hat{\beta}$ can be conducted using likelihood ratio (LR) tests with standard $\chi^2$ inference. Let the form of the linear restrictions on $\beta$ be given by $\beta = H\varphi$ where $H$ is a $p \times s$ matrix of restrictions and $\varphi$ is a $s \times r$ matrix of unknown parameters. The LR test statistic is given by,

$$T \sum_{i=1}^{r} \ln \left[ \frac{(1 - \tilde{\lambda}_i)}{(1 - \hat{\lambda}_i)} \right] \sim \chi^2_{r(p-s)}$$  \hfill (5)

where $\tilde{\lambda}_i$ are the eigenvalues from the restricted MLE.

The VAR in (4) has a Wold moving average representation (MAR) which is given by (6).

$$X_t = \delta_t + C(L)\varepsilon_t$$  \hfill (6)

where $C(L) = B(L)^{-1}$ and $\delta_t = \mu_t B(L)^{-1}$. Equation (6) can be rewritten in common trends form by decomposing $C(L) = C(1)(1 - L)^{-1} + C^\ast(L)$.

$$X_t = \delta_t + C(1) \sum_{s=0}^{t} \varepsilon_s + C^\ast(L)\varepsilon_t$$  \hfill (7)

Johansen (1991) derives sufficient conditions for (7) to exist when $X_t$ is characterized by cointegration. The inversion of the VAR in (4) is not straightforward due to the singularity created by the reduced rank of $\alpha\beta'$. Johansen, following Engle and Granger (1987), demonstrates that $C(1) = \beta_\perp (\alpha_\perp' \Gamma(1)\beta_\perp)^{-1}\alpha_\perp'$, where $\alpha_\perp'\alpha = 0$, $\beta_\perp'\beta = 0$ and $\Gamma(1) = I - \sum_{j=1}^{k-1} \Gamma_j$.
all derived from (4). In (7) $C(1) \sum_{s=0}^{t} \varepsilon_s$ represents the permanent component of $X_t$. From the definition of $C(1)$ a natural interpretation is that $\alpha'_{\perp} \sum_{s=0}^{t} \varepsilon_s$ represents the common stochastic trends and $\beta_{\perp}(\alpha'_{\perp} \Gamma(1)\beta_{\perp})^{-1}$ represents the factor loadings that pass the trends on to the individual elements in $X_t$. Since the common trends are orthogonal to the error correction space, any variable that is weakly exogenous in (4) can be identified as the source of the common trend. Weak exogeneity in this context can be tested by examining the statistical significance of the error correction coefficients in each equations of (4).

4 Empirical Results

4.1 Data

The data used are from the OECD Main Economic Indicators and are monthly observations on short-term nominal interest rates and CPI inflation. The sample covers the period from January 1960 to August 1993. The interest rates and inflation rates are converted to annualized values. The Belgian interest rate used is that on the three-month Treasury certificates. The German interest rate is the FIBOR rate. The French, British and Dutch nominal interest rates are the call money rates. The Italian rate used is the Treasury bond rate with 6-year average maturity. The Japanese and American nominal interest rates are three-month Treasury bill rates. The data used in the study are plotted in figure 1.
4.2 Cointegration Results

Table 1 presents the results from the cointegration analysis for each of the eight countries. The first row of the table designates which column applies to which country. The second row presents the Johansen trace statistics of the null hypothesis that there are zero cointegrating vectors, i.e., $r = 0$. Row three presents marginal significance levels of the test statistics in the second row. Row four displays the trace statistics for the null that $r = 0$. The fifth row presents the p-values associated with the statistics in row four. The sixth row shows the estimated Fisher effect for each country. Rows six, seven and eight display the results of various hypothesis tests on the cointegration space. Rows nine and ten display hypothesis tests on the error correction coefficients. All of the results presented in table 1 come from estimation of (4) with $k = 7$.

The evidence displayed in table 1 strongly supports the existence of cointegration between nominal interest rates and inflation rates in all eight countries. The null hypothesis of zero cointegration vectors, $r = 0$, is rejected for every country at high significance levels. The hypothesis that $r \leq 1$ is not rejected at any conventional significance level for any of the countries examined. Exclusion tests of the cointegration parameters reveal that neither variable can be omitted from the system.

Turning attention to the estimate of the Fisher effect $\hat{\beta}_2$, all of the point estimates are greater than one as implied by the Darby hypothesis. Hypothesis tests on the value of the Fisher effect reject the null hypothesis of unity at the 5% level or higher for all but the U.K. system. This is strong support for the Darby effect.
From the definition of $C(1)$ and the interpretation of $\alpha_{\perp}^t \sum_{s=0}^{t} \varepsilon_s$ as the common trends, hypotheses on $\alpha$ are of interest since they relate directly to the space spanned by $\alpha_{\perp}$. In all eight cases, the error correction coefficient is significant in the nominal interest rate equation but insignificant in the inflation equation of (4). This evidence implies that the inflation rate is the long-run driving force in each of these Fisher relationships.

5 Conclusions

Tests of the Fisher relation have been almost exclusively concentrated on the U.S. The evidence presented in the literature is mixed. While Fisher’s original formulation suggested a one-to-one response of nominal interest rates to changes in expected inflation, Darby (1975) argues that the response of nominal rates should be much greater than one since interest income is subject to taxation. Most estimates of the Fisher effect are below unity, thus violating the Fisher-plus effect advocated by Darby. There are several possible reasons for these low estimates. The first is due to Tobin (1965, 1969). He suggests that investors rebalance their portfolios in favor of real assets when expected inflation is unusually high. A second argument made by Tanzi (1980) implies that investors are irrational and suffer from fiscal illusion. A third explanation comes from Evans and Lewis (1995). They hypothesize that peso problems exist in the market for nominal debt and these will lead researchers to (incorrectly) conclude that agents are behaving in an irrational manner. Finally, Mishkin (1992) asserts that both nominal interest rates and inflation behave as nonstationary variables and thus require appropriate estimation techniques.
I have analyzed the Fisher relation for a set of eight industrialized countries over a sample that spans the last three decades. The evidence clearly supports the Fisher relation in all eight countries and the Darby effect in seven of the eight. Only the U.K. Fisher effect is not statistically greater than one. This result is important since it suggests that specifying the ex-post real interest rate as the simple difference between nominal interest rates and inflation rates is probably a misspecification that may lead to biased estimates.
Notes

1. An exception is Crowder (1997), where the long-run Fisher relation is examined for Canada.

2. It is an excepted proposition among international economists that real interest rates are not equal internationally. Crowder and Chung (2001) present evidence that the source of rejection of real rate equality is uncovered interest parity and ex-ante PPP.

3. In consumption based versions of (1), $q_t$ would be proportional to consumption growth.

4. I am ignoring any common deterministic trends that my appear in this expression for ease of exposition.

5. The Interbank Loan Rate is available from 1971 till the end of the sample. I compared the results from using this interest rate series with those obtained by using the 6-year Treasury notes with no qualitative difference in results.

6. The Japanese interest rate series was obtained from the Bank of Japan.

7. These p-values were generated from the program JOHDIST.EXE distributed by James MacKinnon to accompany MacKinnon et al. (1996).

8. This was the optimal lag for all but three using the AIC. For the other three a shorter lag was determined to be optimal based upon AIC. Since underfitting the statistical model has more dire consequences for inference than overfitting, which results in a loss of efficiency, I chose to use $k = 7$. As it turns out, the results are relatively insensitive to alternative lag selections. I fit (3) with $k = 5, 9, 11$ and 13 with no change in the qualitative results.

9. These tests can be interpreted as multivariate unit root tests on the non-excluded vari-
able. Under the assumption of one cointegrating vector \( \beta = [\beta_1, -\beta_2]' \) if \( \beta_1 \) is not statistically different from zero then the second variable is stationary in levels.

\(^{10}\)The estimate of \( \beta, \hat{\beta} \), is normalized on \( \hat{\beta}_1 \) such that \( \hat{\beta} = [1, -\hat{\beta}_2]' \). Thus \( \hat{\beta}_2 \) can be interpreted as \( a_1 \) from (2).
References


Table 1: Johansen Cointegration Analysis of Fisher Relationships

<table>
<thead>
<tr>
<th>Country</th>
<th>USA</th>
<th>UK</th>
<th>Japan</th>
<th>Germany</th>
<th>France</th>
<th>Italy</th>
<th>Netherlands</th>
<th>Belgium</th>
</tr>
</thead>
<tbody>
<tr>
<td>trace (H_0 : r = 0)</td>
<td>19.256</td>
<td>22.858</td>
<td>38.531</td>
<td>65.353</td>
<td>14.686</td>
<td>26.865</td>
<td>29.598</td>
<td>16.631</td>
</tr>
<tr>
<td>P-value</td>
<td>0.03</td>
<td>0.01</td>
<td>&gt; 0.01</td>
<td>&gt; 0.01</td>
<td>0.02</td>
<td>&gt; 0.01</td>
<td>&gt; 0.01</td>
<td>0.09</td>
</tr>
<tr>
<td>trace (H_0 : r \leq 1)</td>
<td>0.681</td>
<td>0.651</td>
<td>0.873</td>
<td>1.186</td>
<td>0.694</td>
<td>0.477</td>
<td>1.153</td>
<td>0.347</td>
</tr>
<tr>
<td>P-value</td>
<td>0.47</td>
<td>0.48</td>
<td>0.41</td>
<td>0.32</td>
<td>0.46</td>
<td>0.55</td>
<td>0.33</td>
<td>0.62</td>
</tr>
<tr>
<td>(\beta_2)</td>
<td>1.310</td>
<td>1.176</td>
<td>1.283</td>
<td>1.720</td>
<td>1.393</td>
<td>1.255</td>
<td>1.430</td>
<td>1.7564</td>
</tr>
<tr>
<td>(H_0 : \beta_i = 0)</td>
<td>15.282</td>
<td>15.596</td>
<td>27.260</td>
<td>51.446</td>
<td>11.020</td>
<td>22.057</td>
<td>18.176</td>
<td>11.888</td>
</tr>
<tr>
<td>(H_0 : \beta_\pi = 0)</td>
<td>17.891</td>
<td>21.543</td>
<td>36.770</td>
<td>62.915</td>
<td>13.299</td>
<td>25.905</td>
<td>27.071</td>
<td>15.938</td>
</tr>
<tr>
<td>(H_0 : \beta = [1, -1]')</td>
<td>6.150</td>
<td>1.490</td>
<td>6.382</td>
<td>40.924</td>
<td>5.711</td>
<td>6.526</td>
<td>5.703</td>
<td>9.934</td>
</tr>
<tr>
<td>(t(\alpha_i = 0))</td>
<td>-4.03</td>
<td>-3.15</td>
<td>-3.97</td>
<td>-3.38</td>
<td>-2.92</td>
<td>-4.92</td>
<td>-2.77</td>
<td>-2.74</td>
</tr>
<tr>
<td>(t(\alpha_\pi = 0))</td>
<td>-0.86</td>
<td>0.89</td>
<td>1.41</td>
<td>0.77</td>
<td>-1.17</td>
<td>0.01</td>
<td>0.63</td>
<td>1.05</td>
</tr>
</tbody>
</table>

**Note:** Results obtained from VAR with 7 lags. The constant term was restricted to equal zero in each cointegrating vector. \(\hat{\beta}_2\) is the estimated Fisher effect. \(H_0 : \beta_i = 0\) is the LR test that the nominal interest rate can be excluded from the cointegrating vector. Under the null of one cointegrating vector this represents a test of the stationarity of \(\pi\). \(H_0 : \beta_\pi = 0\) is the LR test that the inflation rate can be excluded from the cointegrating vector. Under the null of one cointegrating vector this represents a test of the stationarity of \(i\). \(H_0 : \beta = [1, -1]'\) is the LR test of that the Fisher effect is equal to one. \(t(\alpha_i = 0)\) is the \(t\)-value of the error correction term in the nominal interest rate equation of the VECM. \(t(\alpha_\pi = 0)\) is the \(t\)-value of the error correction term in the inflation equation of the VECM.
Figure 1