Review Problems 2

Forward z Transform Problems

1. Find z-transforms of the following in closed form. For sequences containing x(), find the transforms in terms of X(), using real coefficients only.
   (a) n u(n)      (b) c^n u(n)      (c) δ(n+8) + δ(n-8)      (d) cos(wo n) u(n)

2. Find z-transforms of the following in closed form, and their regions of convergence.
   (a) u(-n)     (b) a^n u(n-5) (c) n u(-n) (d) cos(w_z n) u(n)

3. Find z-transforms of the following sequences in closed form, and their regions of convergence. For sequences containing x(), find the transforms in terms of X(), using real coefficients only. Assume the R.O.C. of X(z) is \(|z| > 0.5\).
   (a) u(5-n)     (b) (n-5) u(n-5) (c) c^n x(n)      (d) n^2 x(n) (e) (0.5/n!) - [(j)^n - (-j)^n] u(n)

4. Find z-transforms of the following sequences in closed form, and their regions of convergence. For sequences containing x(), find the transforms in terms of X(), using real coefficients only. Assume the R.O.C. of X(z) is \(|z| > 0.5\).
   (a) u(n-7)     (b) n x(-n) (c) c^n u(n)      (d) cos(n/3) u(n) (e) (0.5/n!) - [(j)^n + (-j)^n] u(n)

5. Find z-transforms of the following sequences in closed form, and their regions of convergence. Assume the R.O.C. of X(z) is \(|z| > 0.3\).
   (a) u(n+9)     (b) -n x(n) (c) c^n u(-n)      (d) δ(n+8) + δ(n-9) (e) n · (c^n/n!) u(n)

6. Find z-transforms of the following sequences in closed form, and their regions of convergence. For sequences containing x(), find the transforms in terms of X(), using real coefficients only. Assume the R.O.C. of X(z) is \(|z| > 0.5\).
   (a) u(5-n)     (b) (n-5) u(n-5) (c) c^n x(n)      (d) n^2 x(n) (e) (0.5/n!) - [(j)^n - (-j)^n] u(n)

7. Find z-transforms of the following sequences in closed form, and their regions of convergence. Assume the R.O.C. of X(z) is \(|z| > 0.3\).
   (a) u(-n)     (b) x(-n) (c) n x(-n) (d) a^n u(n) (e) cos(-w_z n) u(n)
Z Transform Properties

1. A time domain sequence $x(n)$ has the $z$-transform $X(z)$.
   (a) Find the $z$-transform of $y(n) = x(2n)$ in terms of $X()$.
   (b) What is the period of $Y(e^{jw})$ ?
   (c) If the frequency response of $x(n)$ is 1, for $|w| \leq \pi/3$, sketch $Y(e^{jw})$ which is the frequency response of $y(n)$.

2. A stable impulse response $h(n)$ has the transfer function $H(z)$, which has the R.O.C. $|z| > .5$
   (a) Does the frequency response of $h(n)$ exist ? If so, give it in terms of $H()$.
   (b) Find the $z$-transform of $h(-n)$ in terms of $H()$, and its R.O.C.
   (c) Does $h(-n)$ have a frequency response ? If so, give it.

3. Let $x(n) = 2^n[u(n) – u(n-N)]$, which is nonzero between $n=0$ and $n=N-1$.
   (a) Find $X(z)$ and its R.O.C. Remember that $x(n)$ is a causal, finite length sequence.
   (b) Find $X(k)$, the DFT of $x(n)$.
   (c) Multiply $X(z)$ by $(-.5z)/(-.5z)$ and give the new $X(z)$ and R.O.C. Is the R.O.C. the same as in part (a) ?
   (d) If we find $x(n)$ from the $X(z)$ of part (c ), is it the same as in the problem statement ? Why ?
Transfer Functions, and Their Relationship to Stability and Causality

1. Given the transfer function

\[ H(z) = \frac{2 - 7z^{-1}}{1 - 7z^{-1} + 10z^{-2}} \]

(a) Give the poles of \( H(z) \), and its region of convergence. Are the poles inside the region of convergence?

(b) Give the region of convergence of \( H(z) \), when \( h(n) \) is anticausal. Are the poles inside the region of convergence?

(c) How many versions of \( h(n) \) can be found from \( H(z) \), each corresponding to a different region of convergence? How many of these versions have a frequency response?

2. An IIR digital filter has the transfer function

\[ H(z) = \frac{2 - 7z^{-1}}{1 - 7z^{-1} + 10z^{-2}} \]

(a) Using the partial fraction expansion, or another method, find the causal impulse response \( h(n) \), as the sum of two exponentials.

(b) Give a stable version of the impulse response \( h(n) \).

(c) Letting \( H(z) = H(z)/D(z) \), where \( D(z) \) is the z-transform of \( \delta(n) \), give a causal difference equation that calculates \( h(n) \) (as in part (a)) in terms of \( \delta(n) \). Give \( D(z) \).

(d) Give the first two terms of \( h(n) \) (\( h(0) \) and \( h(1) \)) that result when long division is used to find the causal \( h(n) \).

(e) Give the first two terms of \( h(n) \) (\( h(-1) \) and \( h(-2) \)) that result when long division is used to find the anti-causal \( h(n) \) from

\[ H(z) = \frac{-7z + 2z^2}{10 - 7z + z^2} \]
3. An IIR digital filter has the unit pulse response

\[ h(n) = (0.3)^n u(n) - (4)^{-n} u(-n) \]

(a) Find \( H(z) \) in closed form, and its R.O.C.
(b) Give the poles of \( H(z) \).
(c) Change \( h(n) \) to make it stable, without changing \( H(z) \), except for its R.O.C.
    Give the new R.O.C. for \( H(z) \)
(d) Give a stable set of recursive difference equations for the filter. Use the parallel form.
(e) Can the difference equations in part (d) be combined into a single stable difference equation? If so, give that difference equation.

4. An IIR digital filter has the unit pulse response

\[ h(n) = (0.6)^n u(n) + (2)^n u(n) \]

(a) Find \( H(z) \) in closed form, and its R.O.C.
(b) Give the poles of \( H(z) \).
(c) Change \( h(n) \) to make it stable, without changing \( H(z) \), except for its R.O.C.
    Give the new R.O.C. for \( H(z) \)
(d) Continuing part (c), write \( H(z) \) as \( H_1(z) + H_2(z) \) such that long divisions, when applied to each of the terms, generates the stable \( h(n) \) as \( h_1(n) + h_2(n) \).
(e) Can the stable filter be applied using a single stable difference equation? If so, give that difference equation.

5. An IIR digital filter has the impulse response

\[ h(n) = (3)^n u(n) + (5)^n u(n) \]

(a) Find \( H(z) \) in closed form, and its region of convergence.
(b) Give the poles of \( H(z) \).
(c) Change \( h(n) \) to make it stable, without changing \( H(z) \). Give the new R.O.C. for \( H(z) \)
(d) Give a stable set of recursive difference equations for the filter. Use the parallel form.
(e) In the pseudocode below, which is based upon part (d), give correct expressions for \( A, B, C, D \) and \( E \). Assume that allowable arguments for \( x(), y(), y_1(), \) and \( y_2() \) include only the integers from 0 to \( N \).
\[ y_1(N) = A \]
\[ y_2(N) = A \]
\[ y(N) = y_1(N) + y_2(N) \]

For \( n = B \) to \( C \)

\[ y_1(n) = D \]
\[ y_2(n) = E \]
\[ y(n) = y_1(n) + y_2(n) \]
End

6. An IIR digital filter has the transfer function

\[ H(z) = \frac{3 \frac{29}{7} z^{-1}}{1 - \frac{15}{7} z^{-1} + \frac{2}{7} z^{-2}} \]

(a) Give the poles of \( H(z) \).
(b) Find the causal impulse response \( h(n) \), which is unstable .
(c) Change \( h(n) \) to make it stable, without changing \( H(z) \). Give \( H(z) \)’s new R.O.C.
(d) Give a stable set of recursive difference equations for the filter. Use parallel form.
(e) Can we combine the difference equations from part (d) into a stable direct form difference equation ?

7. An IIR digital filter has the transfer function

\[ H(z) = \frac{1}{1 - 3z^{-1}} + \frac{1 - \frac{1}{2} \cos(1) z^{-1}}{1 - \cos(1) z^{-1} + \frac{1}{4} z^{-2}} \]

(a) Find a causal impulse response \( h(n) \) if the R.O.C. for \( H(z) \) is the exterior of a circle in the complex \( z \) plane.
(b) Give the poles of \( H(z) \).
(c) Change \( h(n) \) in part (a) to make it stable, without changing \( H(z) \) , except for its R.O.C. Give the new R.O.C. for \( H(z) \)
(d) Give a stable set of recursive difference equations for the filter. Use the parallel form. The coefficients must be real.
(e) Can the difference equations in part (d) be combined into a single stable difference equation ? If so, give that difference equation.
8. An IIR digital filter has the impulse response

\[ h(n) = \sum_{i=1}^{N} c_i \cdot a_i^n u(n) \]

(a) Find \( H(z) \) in closed form and give the region of convergence.
(b) Assuming that the mth term of \( h(n) \), \( c_m(a_m)^n u(n) \), is unstable, give a new version of the mth term that is stable and has the same \( z \)-transform, but with a different R.O.C.
(c) Given \( H(z) \) in part (a), with no R.O.C., how many different \( h(n) \) sequences can be formed, at most, simply by changing the causality and R.O.C. of the different terms?
(d) Given \( H(z) \) in part (a), with no R.O.C., assume that we can replace \( z \) by \( e^{jw} \) to obtain a frequency response. How many versions of \( H(z) \) at most, each with a different R.O.C., can have this frequency response?
(e) Let \( H_m(z) \) be the mth term in \( H(z) \) in part (a), which has input \( X(z) \) and output \( Y_m(z) \). Give a stable difference equation for \( H_m(z) \) if \( h_m(n) \) is causal and stable.
(f) Repeat part (e) if \( h_m(n) \) is causal and unstable.
Inverse z Transform

1. \( Y(z) = \exp(az - bz^{-1}) \), where “a” and “b” are real, positive, finite, nonzero.

(a) The signal \( y(n) \) can be expressed as the convolution \( c(n)*a(n) \) where \( c(n) \) is causal and \( a(n) \) is anticausal. Without using delta functions, give expressions for both \( c(n) \) and \( a(n) \).

(b) Find \( a(0) \) and \( c(0) \).

(c) Give expressions for \( a(n) \) in terms of \( a(n+1) \), and \( c(n) \) in terms of \( c(n-1) \).

(d) In the pseudocode below for calculating \( a(n) \) and \( b(n) \), give expressions for \( A \) and \( B \), modifying your work from part (c), if necessary.

\[
\begin{align*}
a(0) &= A \\
c(0) &= B \\
For \ n = 1 \ to \ 999 \\
c(n) &= C \\
a(-n) &= D \\
End
\end{align*}
\]

(e) Give expressions for \( C \) and \( D \), modifying your work from part (c), if necessary.

2. \( Y(z) = \exp(c(z-z^{-1})) \), where \( c \) is real.

(a) Using Euler’s formula, find a simplified expression for \( Y(e^{jw}) \)

(b) Give an expression for \( \text{Re}\{ Y(e^{jw}) \} \). Is it an even function of \( w \)?

(c) Give an expression for \( \text{Im}\{ Y(e^{jw}) \} \). Is it an odd function of \( w \)?

(d) Based upon your answers in parts (b) and (c), state whether \( y(n) \) is real, complex, or imaginary.

(e) Based upon your answers in parts (b) and (c), state whether \( y(n) \) is even, odd, or neither.

(f) \( y(n) = a(n)*d(n) \) where \( a(n) \) is anti-causal and \( d(n) \) is causal. Give \( d(n) \) without the use of delta functions.

3. \( Y(z) = \exp(b(z+z^{-1})) \), where \( b \) is real.

(a) Using Euler’s formula, find a simplified expression for \( Y(e^{jw}) \)

(b) Give an expression for \( \text{Re}\{ Y(e^{jw}) \} \). Is it an even function of \( w \)?

(c) Give an expression for \( \text{Im}\{ Y(e^{jw}) \} \). Is it an odd function of \( w \)?

(d) Based upon your answers in parts (b) and (c), state whether \( y(n) \) is real, complex, or imaginary.

(e) Based upon your answers in parts (b) and (c), state whether \( y(n) \) is even, odd, or neither.

(f) \( y(n) = a(n)*d(n) \) where \( a(n) \) is anti-causal and \( d(n) \) is causal. Give \( d(n) \) without the use of delta functions.
4. \( Y(z) = \exp(c(z-z^{-1})) \), where \( c \) is real.

(a) Using Euler’s formula, find a simplified expression for \( Y(e^{jw}) \)

(b) Give an expression for \( \text{Re}\{Y(e^{jw})\} \). Is it an even function of \( w \) ?

(c) Give an expression for \( \text{Im}\{Y(e^{jw})\} \). Is it an odd function of \( w \) ?

(d) Based upon your answers in parts (b) and (c), state whether \( y(n) \) is real, complex, or imaginary.

(e) Based upon your answers in parts (b) and (c), state whether \( y(n) \) is even, odd, or neither.

(f) \( y(n) = a(n)^*d(n) \) where \( a(n) \) is anti-causal and \( d(n) \) is causal. Give \( d(n) \) without the use of delta functions.
Inverse DFT, Parseval’s Equation

1. Find $y(n)$ in terms of $x(n)$ and $h(n)$ (which can be complex).

   \( a) \ Y(k) = H^*(-k)_N \cdot X(k) \quad b) \ Y(k) = \sum_{m=0}^{N-1} H(m + k)_N \cdot X(m + 2k)_N \)

2. Find $y(n)$ in terms of $x(n)$ and $h(n)$ (which can be complex).

   \( a) \ Y(k) = H^*(k) \cdot X(k), \quad b) \ Y(k) = \sum_{m=0}^{N-1} H(m - k)_N \cdot X(m + k)_N \)

3. Find $y(n)$ in terms of $x(n)$ and $h(n)$ (which can be complex) if

   \( a) \ Y(k) = H^*(k) \cdot X^*(k) \quad b) \ Y(k) = \sum_{m=0}^{N-1} H(m)_N \cdot X(m + 2k)_N \)

4. Find $y(n)$ in terms of $x(n)$ and $h(n)$ (which can be complex) if

   \( a) \ Y(k) = H^*(k) \cdot X(k) \quad b) \ Y(k) = \sum_{m=0}^{N-1} H(m + k)_N \cdot X(m + 4k)_N \)

5. Find $y(n)$ in terms of $x(n)$ and $h(n)$ (which can be complex) if

   \( a) \ Y(k) = H^*(k) \cdot X^*(k) \quad b) \ Y(k) = \sum_{m=0}^{N-1} H(m - k)_N \cdot X(m - 2k)_N \)

6. Find $y(n)$ in terms of $x(n)$ and $h(n)$ (which can be complex) if

   \( a) \ Y(k) = H^*(k) \cdot X(k) \). Give the expression that should be substituted for $H^*(k)$.

   \( b) \ Y(k) = \sum_{m=0}^{N-1} H(m - k)_N \cdot X(m - 3k)_N \). What substitution should first be performed to simplify this expression?

7. Find $y(n)$ in terms of $x(n)$ and $h(n)$ (which can be complex) if

   \( a) \ Y(k) = H^*(k) \cdot X^*(k) \quad b) \ Y(k) = \sum_{m=0}^{N-1} H(m)_N \cdot X(m + 2k)_N \)
8. Find $y(n)$ in terms of $x(n)$ and $h(n)$ (which can be complex). $L$ and $M$ are integers.

\[(a) \, Y(k) = X^*(k) \cdot H(-k) \quad (b) \, Y(k) = \sum_{m=0}^{N-1} H(m + L \cdot k) \cdot X(m + M \cdot k)\]
Filter Design Using Inverse DFT

1. A filter is needed to recover $x_1(n)$ from the signal, $x(n) = x_1(n) + x_2(n) + x_3(n)$, where $x_1(n) = \sin(0.5n)$, $x_2(n) = \sin(1.4n)$, and $x_3(n) = \cos(2.7n)$.
   (a) What kind of filter is required, LP, BP, HP, or BR?
   (b) Specify ranges for the filter's cut-off frequency or frequencies $w_i$ in radians, for $i = 1, 2, ..., (There is either 1 cut-off frequency, or 2)
   (c) Specify cut-off sample or samples, $k_1$ etc, for $H(k)$, in terms of the cut-off frequencies $w_i$.
   (d) Find a closed form for the impulse response $h(n)$ using $H(k)$, the inverse DFT, and the cut-off sample symbol(s) $k_i$.

2. A filter is needed to recover $x_2(n)$ from the signal, $x(n) = x_1(n) + x_2(n) + x_3(n)$, where $x_1(n) = \sin(0.5n)$, $x_2(n) = \sin(1.4n)$, and $x_3(n) = \cos(2.7n)$.
   (a) What kind of filter is required, LP, BP, HP, or BR?
   (b) Specify a filter cut-off frequency or frequencies in radians, and find the corresponding cut-off sample or samples, $k_1$ etc, for $H(k)$.
   (c) Assume that $H(k)$ is ideal in the sense that each of its samples is either 1 or 0. Find a closed form for the impulse response $h(n)$ using $H(k)$, the inverse DFT operation, and the cut-off sample symbol(s) $k_i$.
   (d) Find a real expression for $h(n)$ from part (c).

3. A filter is needed to recover $x_1(n)$ from the signal, $x(n) = x_1(n) + x_2(n) + x_3(n)$, where $x_1(n) = \sin(0.5n)$, $x_2(n) = \sin(1.4n)$, and $x_3(n) = \cos(2.7n)$.
   (a) What kind of filter is required, LP, BP, HP, or BR?
   (b) Specify ranges for the filter's cut-off frequency or frequencies $w_i$ in radians, for $i = 1, 2, ..., (There is either 1 cut-off frequency, or 2)
   (c) Specify cut-off sample or samples, $k_1$ etc, for $H(k)$, in terms of the cut-off frequencies $w_i$.
   (d) Find a closed form for the impulse response $h(n)$ using $H(k)$, the inverse DFT, and the cut-off sample symbol(s) $k_i$. 
4. A causal FIR digital filter $h(n)$ is to be designed using the inverse DFT. Assume that its desired frequency response $H_d(e^{jw})$ is available for $0 \leq w \leq \pi$.
   (a) When generating $H(k)$, $H_d(e^{jw})$ is to be sampled as $w = k\cdot\Delta w$, where $\Delta w$ is known.
      Find the DFT order $N$ in terms of $\Delta w$.
   (b) The pseudocode below uses the inverse DFT or FFT to generate $h(n)$. Give the correct value for $W$, in the first line of code.
   (c) Give the correct expression for $X$, in terms of $N$, in the second line of code.
   (d) Give $Y$, in the third line of code, using $N$ rather than $\Delta w$.
   (e) Give the correct expression for $Z$, in the fifth line of code.

   $H(0) = H(exp(jW))$
   For $1 \leq k \leq X$
   $w(k) = Y$
   $H(k) = H(exp(jw(k)))$
   $H(N-k) = Z$
   End
   $h(n) = DFT^{-1}\{H(k)\}$

5. A causal FIR digital filter $h(n)$ is to be designed using the inverse DFT. Assume that its desired frequency response $H_d(e^{jw})$ is available for $0 \leq w \leq \pi$.
   (a) When generating $H(k)$, $H_d(e^{jw})$ is to be sampled at frequencies $w(k) = k\cdot\Delta w$.
      Assuming $N$ is known, find $\Delta w$ in terms of the DFT order $N$.
   (b) The pseudocode below uses the inverse DFT or FFT to generate $h(n)$. Give the correct value for $W$, in the first line of code.
   (c) Give the correct expression for $X$, in terms of $N$, in the second line of code.
   (d) Give $Y$ in the third line of code, using $N$ rather than $\Delta w$.
   (e) Give the correct expression for $Z$, in the fifth line of code.
   (f) Assuming that the filter will have $N_1$ coefficients, find the values for $A$ and $B$ in the third to last line.

   $H(0) = H_d(exp(jW))$
   For $1 \leq k \leq X$
   $w(k) = Y$
   $H(k) = H_d(exp(jw(k)))$
   $H(N-k) = Z$
   End

   $h(n) = DFT^{-1}\{H(k)\}$

   For $A \leq n \leq B$
   $h(n) = 0$
   End
Advanced Material

1. A signal \( x(n) \) has an infinite number of samples and is to be convolved with a causal filter \( h(n) \), which is nonzero for \( 0 \leq n \leq N_1-1 \). The output is \( y(n) \).
   (a) The convolution can be done efficiently as follows. Let \( x_m(n) = x(n+(m-1)\cdot N_2) \) for \( 0 \leq n \leq N_2-1 \) and \( x_m(n) = 0 \) for \( N_2 \leq n \leq N_3-1 \). Here, \( m \) varies from 1 to infinity. The impulse response array \( h(n) \) is padded with zeroes so that \( h(n) = 0 \) for \( N_1 \leq n \leq N_3-1 \). Now, \( Y_m(k) = H(k) \cdot X_m(k) \) and \( y_m(n) = \text{DFT}^{-1}\{ Y_m(k) \} \). Give the smallest possible DFT order, \( N_3 \).
   (b) Give the delay \( N_5 \) so that \( y(n) \) from part (a) can be constructed as
   \[
   y(n) = \sum_{m=1}^{\infty} y_m(n-(m-1)\cdot N_5)
   \]
   (c) The filter \( h(n) \) is symmetric (even or odd) about sample number \( N_6 \) (so \( h(N_6-k) = h(N_6+k) \) for example). Since \( y(n) \) is produced one block at a time, some samples are delayed more than others. Give the minimum and maximum time delays for samples of \( y(n) \), if \( y(n) \) samples are output a block at a time.
   (d) Give the minimum allowable value for \( N_2 \).

2. For the infinite length sequence \( x(n) \), we want to calculate a frequency domain model in a moving N-sample window as
   \[
   X_n(k) = \sum_{m=0}^{N-1} x(n + m) \cdot W_N^{mk}
   \]
   (a) Express \( X_{n+1}(k) \) as \( X_n(k)W_N^{-k} \) plus two additional terms.
   (b) Replacing \( n \) by \( n-1 \) in part (a), give a final expression for \( X_n(k) \) in terms of \( X_{n-1}(k) \)
   (c) Assume \( x(n) = 0 \) for \( n \) negative. What is the smallest value of \( n \) for which \( X_n(k) \) in part (b) is calculated using no zero-valued samples. (Hint: \( X_n(k) = [X_{n-1}(k) - x() + x()]W_N^0 \). All 3 terms in the brackets have \( x() \).)
   (d) Before calculating \( X_n(k) \) for \( n=0 \), how should \( X_{n-1}(k) \) be initialized?
   (e) For \( n \) varying from 0 to \( N-2 \), modify your answer from part (b), so that no zero-valued quantities are used.
3. For the infinite length sequence \( x(n) \), we want to calculate a frequency domain model in a moving \( N \)-sample window as

\[
X_n(k) = \sum_{m=0}^{N-1} x(f(n)+m) \cdot W_N^{mk}
\]

(a) Find \( f(n) \) so that as \( m \) varies from 0 to \( N-1 \), \( f(n)+m \) varies from \( n-(N-1) \) to \( n \).
(b) Using your result from (a), and replacing \( n \) by \( (n+1) \), give an expression for \( X_{n+1}(k) \). Do not use the symbol \( f() \).
(c) Express \( X_{n+1}(k) \) as \( X_n(k)W_N^{-k} \) plus two additional terms.
(d) Replacing \( n \) by \( n-1 \) in (c), give a final expression for \( X_n(k) \) in terms of \( X_{n-1}(k) \)

4. For the infinite length sequence \( y(n) \), we want to calculate a frequency domain model in a moving \( N \)-sample window as

\[
Y_n(k) = \sum_{m=0}^{N-1} y(n+m) \cdot W_N^{mk}
\]

(a) Express \( Y_{n-1}(k) \) as \( Y_n(k)W_N^{-k} \) plus two additional terms.
(b) Replacing \( n \) by \( n+1 \) in part (a), give a final expression for \( Y_n(k) \) in terms of \( Y_{n+1}(k) \)

5. Let \( X(e^{jw}) = \text{DTFT}\{x(n)\} \) and let \( x(n) = \text{DFT}^{-1}\{X(k)\} \), where \( x(n) \) is nonzero between \( n=0 \) and \( n=M-1 \). Substituting the 2\(^{nd} \) equation into the first one, we want to find an expression for \( X(e^{jw}) \) in terms of \( X(k) \), such that

\[
X(e^{jw}) = \sum_{k=0}^{N-1} a(k) \cdot X(k)
\]

(a) Give \( a(k) \) in closed form as a function of \( w \) and \( W_N \). Do not substitute 1 for \( W_N \).
(b) Re-write \( a(k) \) using the exponential form of \( W_N \) and the symbol \( w(k) \).
(c) Re-write \( a(k) \) from part (b) as an exponential times a sine divided by a sine.
(d) Given that \( w \) is not a DFT frequency, and given the fact that \( a(k) \) acts like a shifted sinc function, find \( k_1 \) so that \( a(k_1) \) has the largest magnitude. Give a second value, \( k_2 \), so that the magnitude of \( a(k_2) \) is also large.
(e) If \( a(k_1) \) and \( a(k_2) \) are the only significant values of \( a(k) \), and if \( k_1 < k_2 \), give an efficient approximation of the expression for \( X(e^{jw}) \).