1. Find z-transforms of the following sequences in closed form, and their regions of convergence. For sequences containing x(), find the transforms in terms of X(), using real coefficients only. Assume the R.O.C. of X(z) is |z| > .5.
   (a) u(n-5)    (b) n·u(n-5)   (c) c^n·x(n)    (d) n²·x(n)    (e) (.5/n!)-[(j)^n+(-j)^n]·u(n)

2. An IIR digital filter has the impulse response

   $$h(n) = (3)^n·u(n) + (5)^n·u(n)$$

   (a) Find H(z) in closed form, and its region of convergence.
   (b) Give the poles of H(z).
   (c) Change h(n) to make it stable, without changing H(z). Give the new R.O.C. for H(z)
   (d) Give a stable set of recursive difference equations for the filter. Use the parallel form.
   (e) In the pseudocode below, which is based upon part (d), give correct expressions for A, B, C, D and E. Assume that allowable arguments for x(), y(), y_1(), and y_2() include only the integers from 0 to N.

   ```
   y_1(N) = A
   y_2(N) = A
   y(N) = y_1(N) + y_2(N)
   For n = B to C
   y_1(n) = D
   y_2(n) = E
   y(n) = y_1(n) + y_2(n)
   End
   ```

3. Y(z) = exp(a(z⁻¹+z)), where “a” is real.
   (a) Find simplified expressions for Y(e^{jw}), Re{ Y(e^{jw})}, and Im{ Y(e^{jw})}. These expressions should not include the symbol jw.
   (b) State whether y(n) is even, odd, or neither.
   (c) State whether or not y(n) is real, complex, or imaginary.
   (d) The signal y(n) can be expressed as the convolution c(n)*b(n) where c(n) is causal and b(n) is anticausal. Without using delta functions, give expressions for both c(n) and b(n).
4. Find $y(n)$ in terms of $x(n)$ and $h(n)$ (which can be complex) if

(a) $Y(k) = H^*(-k) \cdot X(k)$  
(b) $Y(k) = \sum_{m=0}^{N-1} H(m+k) \cdot X(m+2k)$

5. A filter is needed to recover $x_1(n)$ from the signal, $x(n) = x_1(n) + x_2(n) + x_3(n)$, where $x_1(n) = \sin(0.5n)$, $x_2(n) = \sin(1.4n)$, and $x_3(n) = \cos(2.7n)$.

(a) What kind of filter is required, LP, BP, HP, or BR?
(b) Specify a filter cut-off frequency or frequencies in radians, and find the corresponding cut-off sample or samples, $k_1$ etc., for $H(k)$.
(c) Assume that $H(k)$ is ideal in the sense that each of its samples is either 1 or 0.
Find a closed form for the impulse response $h(n)$ using $H(k)$, the inverse DFT, and the cut-off sample symbol(s) $k_i$.
(d) Find a real expression for $h(n)$ from part (c).

6. A causal FIR digital filter $h(n)$ is to be designed using the inverse DFT. Assume that its desired frequency response $H_d(e^{jw})$ is available for $0 \leq w \leq \pi$.

(a) When generating $H(k)$, $H_d(e^{jw})$ is to be sampled as $w = k \cdot \Delta w$, where $\Delta w$ is known. Find the DFT order $N$ in terms of $\Delta w$.
(b) The pseudocode below uses the inverse DFT or FFT to generate $h(n)$. Give the correct value for $W$, in the first line of code.
(c) Give the correct expression for $X$, in terms of $N$, in the second line of code, assuming that $N$ is odd.
(d) Give the correct expression for $Y$, in the third line of code, using $N$ rather than $\Delta w$.
(e) Give the correct expression for $Z$, in the fifth line of code.

```
H(0) = H(exp(jW))
For 1 \leq k \leq X
w(k) = Y
H(k) = H(exp(jw(k)))
H(N-k) = Z
End
h(n) = DFT^{-1}[H(k)]
```