Exam 1, EE5350, Fall 2014

1. Convolve $h(n)$ and $x(n)$ to get $y(n)$. Put $y(n)$ in closed form when possible.
   (a) $h(n) = 3^n u(n)$ and $x(n) = 2^n u(n)$.
   (b) $h(n) = u(n+6) - u(n-7)$, $x(n) = u(n+2) - u(n-8)$. Express the result in terms of $r(n)$, where $u(n)*u(n) = r(n+1)$.
   (c) $h(n) = (1/2)^n u(n)$, $x(n) = 3^n u(-n)$.
   (d) $h(n) = \sin(w_c n) u(n)$, $x(n) = u(n)$.

2. A system is described by the recursive difference equation
   \[ y(n) = x(n) + y(n - 1) - \frac{2}{9} y(n - 2) \]
   (a) Find $H(e^{jw})$ in closed form. Give $H(e^{j0})$ and $H(e^{j\pi})$.
   (b) Give the homogeneous solution to the difference equation above.
   (c) Re-write the difference equation so that it generates the impulse response $h(n)$.
       Give numerical values for $h(0)$ and $h(1)$.
   (d) Using your answers to parts (b) and (c), give the impulse response $h(n)$.
   (e) The pseudocode below should calculate the magnitude response $|H(e^{jw})|$ for this problem for $M$ frequencies $w(k)$, where $k$ varies from 0 to $M-1$. These frequencies are evenly spaced and include 0 and $\pi$. The complex variables in the pseudocode are $z$, $X_N$, $X_D$ and $H$. Give expressions for $A$, $B$, $C$, and $D$.

\[
\Delta w = A \\
w = -\Delta w \\
\text{For } 0 \leq k \leq M-1 \\
w = w + \Delta w \\
z = e^{jw} \\
X_N = B \\
X_D = C \\
H = X_N/X_D \\
Amp(k) = |H| \\
w(k) = D \\
\text{End}
\]
3. Let \( x(n) \), \( h(n) \) and \( y(n) \) denote complex sequences with DTFTs \( X(e^{jw}) \), \( H(e^{jw}) \) and \( Y(e^{jw}) \). Find frequency domain expressions for the following:

(a) \( C = \sum_{n=-\infty}^{\infty} x(n) \cdot y^*(n) \)

(b) \( y(n) = \sum_{k=-\infty}^{\infty} h(k)x(k-n) \)

(c) Find the numerical value of

\[
\sum_{n=-\infty}^{\infty} \frac{-\sin(\pi n/5)}{6\pi n} \cdot \frac{\sin(\pi n/12)}{4\pi n}
\]

4. The Fourier transform \( X_c(j\Omega) \) of the continuous time signal \( x_c(t) \) has cut-off frequencies of \( \Omega_1 \) and \( \Omega_2 \) respectively. \( X_c(j\Omega) \) is nonzero only between \( \Omega_1 \) and \( \Omega_2 \) and \( \Omega_1 \) may or may not be equal to \( -\Omega_2 \). A C/D converter with sampling period \( T \) samples \( x_c(t) \), producing discrete time signal \( x(n) \). Note that \( x_c(t) \) may be complex.

(a) Give the gain of the filter that reconstructs \( x_c(t) \) from \( X_s(j\Omega) \). Give the filter’s frequency response in terms of step functions \( u(\Omega...) \).

(b) In terms of \( \Omega_1 \) and \( \Omega_2 \), find the maximum sampling period \( T \) so that \( X_s(j\Omega) \) is not aliased.

(c) Give \( X_s(j\Omega) \) in terms of \( x(n) \) and \( T \).

(d) \( x_c(t) \) can be expressed as

\[
x_c(t) = \sum_{n=-\infty}^{\infty} x(n)\Phi(t,T,n,\Omega_1,\Omega_2)
\]

Give an expression for \( \Phi(t,T,n,\Omega_1,\Omega_2) \)

5. Assume that

\[
X(e^{jw}) = e^{-|w|} \quad \text{for} \quad |w| \leq \pi/2, \, 0 \text{ else}
\]

(a) Find \( x(0) \)

(b) Using Parseval’s theorem, find the numerical value of

\[
\sum_{n=-\infty}^{\infty} x^2(n)
\]

(c) Given your answer in part (b), give \( \lim x(n) \) as \( n \) approaches infinity.

(d) Find a real expression for \( x(n) \) and state whether or not it is absolutely summable.
6. A linear, time-invariant filter has the impulse response

\[ h(n) = \begin{cases} 
\frac{1}{n}, & 1 \leq n \leq \infty \\
0, & \text{else} 
\end{cases} \]

(a) Express \( y(n) \) in terms of \( x(n) \) for this filter.
(b) Give the DC gain \( H(e^{j0}) \) in terms of the symbol \( h(n) \).
(c) Using your expression from part (b), find the DC gain \( H(e^{j0}) \) for this filter.
(d) Is this filter causal? Is it BIBO stable?
(e) Is this an FIR or an IIR filter?