ERRATUM: PERTURBATION OF PARTITIONED HERMITIAN DEFINITE GENERALIZED EIGENVALUE PROBLEMS

RENCANG LI†, YUJI NAKATSUKASA‡, NINOSLAV TRUHAR§, AND SHUFANG XU¶

Abstract. The main purpose of this erratum is to correct mistakes in the proof of Theorem 2.4 of [R.-C. Li et al., SIAM J. Matrix Anal. Appl., 32 (2011), pp. 642–663] and in the inequalities (2.23), (2.24), and (2.25) on p. 653 of the same paper.

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For the proof of Theorem 2.4 of [1], line 6 from the bottom of p. 650 of the same paper applied the induction assumption incorrectly. The correct application would have used $\nu_i$ instead of $\tilde{\lambda}_i$. Despite this mistake, the theorem itself remains true. We shall now present a new proof. To do so, we need the following lemma.

LEMMA A (see [2, Lemma 2.1]). Suppose $H - \lambda (S + \Delta S)$ and $H - \mu_i \Delta S - \lambda S$ are Hermitian positive definite pencils. If $\mu_i$ is the $i$th eigenvalue of the first pencil, then it is also the $i$th eigenvalue of the second pencil.

THEOREM 2.4. Under the conditions of Theorem 2.2, we have, for all $1 \leq i \leq N$,

\[
|\tilde{\lambda}_i - \lambda_i| \leq \frac{2\|E_{21} - \tilde{\lambda}_i F_{21}\|_2}{\eta_i + \sqrt{\eta_i^2 + 4\|E_{21} - \tilde{\lambda}_i F_{21}\|_2^2}} \leq \frac{2\|E_{21} - \tilde{\lambda}_i F_{21}\|_2^2}{\eta_i} + \sqrt{\eta_i^2 + 4\|E_{21} - \tilde{\lambda}_i F_{21}\|_2^2}
\]

Proof. Notice that $\tilde{\lambda}_i$ is the $i$th largest eigenvalue of the Hermitian positive definite pencil

\[
\tilde{A} - \lambda \tilde{B} = \begin{pmatrix} A_{11} & E_{21}^* \\ E_{21} & A_{22} \end{pmatrix} - \lambda \begin{pmatrix} I_m & F_{21}^* \\ F_{21} & I_n \end{pmatrix} = \begin{pmatrix} A_{11} & E_{21}^* \\ E_{21} & A_{22} \end{pmatrix} - \lambda \left[I_N + \begin{pmatrix} 0 & F_{21}^* \\ F_{21} & 0 \end{pmatrix}\right]
\]

whose $i$th largest eigenvalue $\tilde{\lambda}_i$, by Lemma A, is the same as that of

\[
\begin{pmatrix} A_{11} & E_{21}^* \\ E_{21} & A_{22} \end{pmatrix} - \tilde{\lambda}_i \begin{pmatrix} 0 & F_{21}^* \\ F_{21} & 0 \end{pmatrix} - \lambda N.
\]
That is to say that $\tilde{\lambda}_i$ is the $i$th largest eigenvalue of the Hermitian matrix

$$
\begin{pmatrix}
A_{11} & E_{21}^* - \tilde{\lambda}_i F_{21}^* \\
E_{21} - \tilde{\lambda}_i F_{21} & A_{22}
\end{pmatrix}.
$$

Now apply Lemma 2.1(c) in [1] to conclude (2.15).

The mistakes in the inequalities (2.23), (2.24), and (2.25) in [1] can be corrected by replacing all $\tilde{\lambda}_j$ in their right-hand sides by $\lambda_j^{(b)}$:

\begin{align}
|\lambda_j^{(b)} - \lambda_j^{(c)}| & \leq \|\tilde{F}_{21}\|_2^2 |\lambda_j^{(b)}| + \|\tilde{E}_{21}\tilde{F}_{21}^* + \tilde{F}_{21}\tilde{E}_{21}^* - \tilde{F}_{21}\tilde{A}_{11}\tilde{F}_{21}^*\|_2 \\
& + \frac{2\|\tilde{E}_{21} - \tilde{F}_{21}\tilde{A}_{11}\|_2^2}{\eta_j + \sqrt{\eta_j^2 + 4}\|\tilde{E}_{21} - \tilde{F}_{21}\tilde{A}_{11}\|_2^2}, \tag{2.23}
\end{align}

\begin{align}
|\lambda_j^{(b)} - \lambda_j^{(c)}| & \leq \frac{2\|\tilde{E}_{21} - \lambda_j^{(b)}\tilde{F}_{21}\|_2^2}{\eta_j + \sqrt{\eta_j^2 + 4}\|\tilde{E}_{21} - \lambda_j^{(b)}\tilde{F}_{21}\|_2^2}, \tag{2.24}
\end{align}

\begin{align}
|\lambda_j^{(b)} - \lambda_j^{(c)}| & \leq \|\tilde{F}_{21}\|_2 \|\lambda_j^{(b)}\| + \frac{2\|\tilde{E}_{21}\|_2^2}{\eta_j + \sqrt{\eta_j^2 + 4}\|\tilde{E}_{21}\|_2^2}. \tag{2.25}
\end{align}

These changes affect Example 2.2 of [1], in which (2.23), (2.24), and (2.25) were used to generate the plots in Figure 2.1 there. Fortunately, these plots have no visible differences from the corresponding ones generated by the corrected (2.23), (2.24), and (2.25) above.

There are several typographical errors in [1]. At line 8 on p. 643, $A$ and $B$ should be $\tilde{A}$ and $\tilde{B}$. At line −9 on p. 643 and also line 20 on p. 654, $\| \cdot \|_F$ should be $\| \cdot \|_F$. At line 14 on p. 645, insert “positive” before “definite.” At line −5 on p. 645, (2.3) should be (2.4). At the end of line −10 on p. 651, (2.18) should be (2.15). At line 5 on p. 653, $Y^*\tilde{A}Y$ and $Y^*\tilde{B}Y$ should be $Y\tilde{A}Y^*$ and $Y\tilde{B}Y^*$.

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REFERENCES