Induced E-field in the D.R.

p-contact

p-type CNR

Exposed Acceptor Ions

Depletion region (DR)

Exposed Donor ions

N-contact

\[ E_x \]

\[ W \]

\[ -X_{pc} \quad -X_p \quad 0 \quad X_n \quad X_{nc} \]
Depletion approx. charge distribution

\[
\begin{align*}
+qN_d &= qN_d x_n [\text{Coul/cm}^2] \\
-qN_a &= -qN_a x_p [\text{Coul/cm}^2]
\end{align*}
\]

Charge neutrality \( \Rightarrow \) \( Q_p' + Q_n' = 0 \), \( \Rightarrow \) \( N_a x_p = N_d x_n \)
Soln to Poisson’s Eq in the D.R.

\[ \frac{dE_x}{dx} = -\frac{qN_a}{\varepsilon} \]

\[ \frac{dE_x}{dx} = \frac{qN_d}{\varepsilon} \]
Comments on the $E_x$ and $V_{bi}$

- $V_{bi}$ is not measurable externally since $E_x$ is zero at both contacts
- The effect of $E_x$ does not extend beyond the depletion region
- The lever rule $[N_a x_p = N_d x_n]$ was obtained assuming charge neutrality. It could also be obtained by requiring $E_x(x=0-\delta x) = E_x(x=0+\delta x) = E_{max}$
Effect of $V > 0$

- Define an external voltage source, $V_a$, with the +term at the p-type contact and the -term at the n-type contact.
- For $V_a > 0$, the $V_a$ induced field tends to oppose $E_x$ due to DR.
- For $V_a < 0$, the $V_a$ induced field tends to add to $E_x$ due to DR.
- Will consider $V_a < 0$ now.
Effect of $V > 0$

The only change now is that

$$x_n - \int E_x \, dx = V_{bi} - V_a,$$

due to $V_a$ tends to reduce $E_x$. Solutions are

$$W = \sqrt{\frac{2\varepsilon(V_{bi} - V_a)}{qN_{\text{eff}}}}$$

and

$$E_{\text{max}} = \sqrt{\frac{2q(V_{bi} - V_a)N_{\text{eff}}}{\varepsilon}}$$
One-sided p+n or n+p jctns

• If $p^+n$, then $N_a >> N_d$, and
  \[ \frac{N_a N_d}{(N_a + N_d)} = N_{\text{eff}} \rightarrow N_d, \text{ and} \]
  \[ W \rightarrow x_n, \text{ DR is all on lightly d. side} \]

• If $n^+p$, then $N_d >> N_a$, and
  \[ \frac{N_a N_d}{(N_a + N_d)} = N_{\text{eff}} \rightarrow N_a, \text{ and} \]
  \[ W \rightarrow x_p, \text{ DR is all on lightly d. side} \]

• The net effect is that $N_{\text{eff}} \rightarrow N^-$, 
  \((- = \text{lightly doped side}) \text{ and } W \rightarrow x_-\)
Junction Capacitance

- The junction has \( +Q'_n = qN_d x_n \) (exposed donors), and (exposed acceptors) \( Q'_p = -qN_a x_p = -Q'_n \), forming a parallel sheet charge capacitor.

\[
Q'_n = qN_d x_n = \frac{qN_d}{N_d} \sqrt{\frac{2\varepsilon(V_{bi} - V_a)(N_a + N_d)}{qN_a N_d}} \\
= \sqrt{\frac{2\varepsilon q(V_{bi} - V_a) N_a N_d}{(N_a + N_d)^2}} \left[ \frac{\text{Coul}}{\text{cm}^2} \right]
\]
Junction
C (cont.)

- This \( Q \sim (V_{bi} - V_a)^{1/2} \) is clearly non-linear, and \( Q \) is not zero at \( V_a = 0 \).
- Redefining the capacitance,

\[
C' \equiv \left| \frac{dQ_n}{dV_a} \right| = \sqrt{\frac{\varepsilon q N_a N_d}{2(V_{bi} - V_a)(N_a + N_d)}}
\]

so \( C' = \frac{\varepsilon A}{W} \), [Fd/cm²], and \( C_j = \frac{\varepsilon A}{W} \), [Fd]
\[ \delta Q_j = \delta Q'_n A \]

\[ +Q'_n = q N_d x_n \]

\[ \delta Q'_n = q N_d \delta x_n \]

\[ Q'_p = -q N_a x_p \]

\[ Q'_p + Q'_n = 0, \Rightarrow N_a x_p = N_d x_n \]
Depletion Capacitance

The capacitance of the depl. region

\[
\frac{\delta Q_j}{\delta V_a} \equiv C_j = \frac{\varepsilon}{W} A,
\]

\[
W = \sqrt{\frac{2\varepsilon(V_{bi} - V_a)}{qN_{eff}}}, \quad N_{eff} \equiv \frac{N_a N_d}{N_a + N_d} \approx N^-
\]

\[
E_{\text{max}} = \sqrt{\frac{2q(V_{bi} - V_a)N_{eff}}{\varepsilon}} = \frac{2(V_{bi} - V_a)}{W}
\]
Junction C (cont.)

• The C-V relationship simplifies to

\[ C_j = C_{j0} \left[ 1 - \frac{V_a}{V_{bi}} \right]^{-\frac{1}{2}}, \text{ a model equation} \]

where \( C_{j0} = \sqrt{\frac{\varepsilon q N_{\text{eff}}}{2V_{bi}}} A, \text{ [Fd]} \)
Junction C (cont.)

- If one plots $[C_j]^{-2}$ vs. $V_a$
  
  Slope = $-[(C_{j0})^2 V_{bi}]^{-1}$
  
  vertical axis intercept = $[C_{j0}]^{-2}$
  
  horizontal axis intercept = $V_{bi}$
Practical Junctions

• Junctions are formed by diffusion or implantation into a uniform concentration wafer. The profile can be approximated by a step or linear function in the region of the junction.

• If a step, then previous models OK.

• If not, $1/2 \rightarrow M$, $1/3 < M < 1/2$. 
Law of the junction (injection of minority carr.)

\[ V_{bi} = V_t \ln \left( \frac{N_a N_d}{n_i^2} \right) = V_t \ln \left( \frac{p_{po}}{p_{no}} \right) = V_t \ln \left( \frac{n_{no}}{n_{po}} \right). \]

Invert to get \[ \frac{p_{no}}{p_{po}} = \frac{n_{po}}{n_{no}} = \exp \left( \frac{- V_{bi}}{V_t} \right), \]

and when \( V_a \neq 0 \), \[ \frac{p_n}{p_p} = \frac{n_p}{n_n} = \exp \left( \frac{V_a - V_{bi}}{V_t} \right). \]
Carrier Injection and diff.

\[ \delta n_p(-x_p) = n_{po} \left( e^{\frac{V_a}{V_t}} - 1 \right) \]

\[ \delta p_n(x_n) = p_{no} \left( e^{\frac{V_a}{V_t}} - 1 \right) \]
Ideal diode equation

- \( I = I_s \left[ \exp\left(\frac{V_a}{nV_t}\right) - 1 \right] \), \( I_s = I_{sn} + I_{sp} \)

**Long diode:** \( W_n \gg L_p \), or \( W_p \gg L_n \)

\[ I_{sn} = qn_i^2 A \frac{D_n}{N_a L_n} , \text{ and } I_{sp} = qn_i^2 A \frac{D_p}{N_d L_p} \]

**Short diode:** \( W_n \ll L_p \), or \( W_p \ll L_n \)

\[ I_{sn} = qn_i^2 A \frac{D_n}{N_a W_p} , \text{ and } I_{sp} = qn_i^2 A \frac{D_p}{N_d W_n} \]
Diffnt'l, one-sided diode conductance

Static (steady-state) diode I-V characteristic

\[ I_D = I_s \left[ \exp \left( \frac{V_a}{nV_t} \right) - 1 \right] \]

\[ g_D \equiv \left[ \frac{dI_D}{dV_a} \right]_{V_Q} \]
References
