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Wireless Sensor Networks

Talk available online at
http://ARRI.uta.edu/acs
ARRI Distributed Intelligence & Autonomy Lab
DIAL

Small mobile Sensor-Dan Popa

Unattended Ground Sensors

Testbed containing MICA2 network (circle), Cricket network (triangle), Sentry robots, Garcia Robots & ARRI-bots (Dan Popa)
Dynamic Localization of Air-Ground Wireless Sensor Network

by

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Objective

• Propose an approach based on potential field method to attain Relative and Absolute Localization.

• Develop a dynamical system to achieve localization. In the approach, the Unattended Ground Sensor (UGS) nodes do not physically move but the virtual dynamics capture the UGS relative position estimates.

• Achieve Air-Ground Localization with on-board GPS on Uninhabited Aerial Vehicles (UAV).
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Introduction

• **What is Localization?**
  – Localization is a mechanism to obtain spatial position estimates of the objects (Unattended Ground Sensor (UGS), mobile platforms, etc.) deployed in an unknown terrain.

• **Why is Localization important?**
  – Target location
  – Map building
  – Border security
  – Pollution monitoring & control
  – Space Exploration

• **Classification of localization**
  – **Relative localization**: In this scheme, the UGS nodes or mobile platforms are localized using the distance measured among the nodes with respect to an arbitrary internal coordinate system.
  – **Absolute localization**: In this scheme, the UGS nodes or mobile platforms are localized with respect to a known coordinate system.
Modeling for Relative Localization

The position estimates for UGS nodes with no absolute position is given as:

\[ X_i = [x_i \ y_i]^T \]

where \( x_i \) and \( y_i \) are the x-y coordinates

The position estimation dynamics are given as:

\[ \ddot{X}_i = \vec{f}_i \]

where \( \vec{f}_i = \begin{bmatrix} f_i^x & f_i^y \end{bmatrix}^T \) is the virtual force in the x and y directions

The State Variable description form for the position estimate is given as:

\[
\begin{bmatrix}
\dot{X}_i \\
\ddot{X}_i
\end{bmatrix} = \begin{bmatrix} O_2 & I_2 \end{bmatrix} \begin{bmatrix} X_i \\
\dot{X}_i
\end{bmatrix} + \begin{bmatrix} O_2 & I_2 \end{bmatrix} \begin{bmatrix} f_i^x \\
f_i^y
\end{bmatrix}
\]

where \( O_2 = \begin{bmatrix} 0 & 0 \\
0 & 0 \end{bmatrix} \) and \( I_2 = \begin{bmatrix} 1 & 0 \\
0 & 1 \end{bmatrix} \)
Choice of Potential Field for Relative Localization

Based on the range information available following potential field is chosen

\[ V = \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{1}{2} K_{ij} (r_{ij} - \bar{r}_{ij})^2 \]

where

\[ r_{ij} = \left[ (x_i - x_j)^2 + (y_i - y_j)^2 \right]^{1/2} \]

\[ \bar{r}_{ij} \] is the measured distance

**Theorem:** Consider the position estimate dynamics \( \dot{X}_i = \bar{f}_i \) for each sensor node in the network. Let the virtual force for \( i^{th} \) sensor node be given as

\[ \bar{f}_i = -\sum_{j=1}^{N} K_{ij} (r_{ij} - \bar{r}_{ij}) \frac{(X_i - X_j)}{\|X_i - X_j\|} - K_v \dot{X}_i \]

Then the position estimates reach steady-state values that provide optimal estimates of the actual relative localization of the nodes in the sense that \( V \) is minimized.
Existence of Local Minima

Define the Lyapunov function as

\[ L = V + \sum_{i=1}^{N} \frac{1}{2} \dot{X}_i^T \dot{X}_i \]

where \( V \) is the potential function defined before.

Due to a nonlinear mapping between the range \((r_{ij})\) and the x-y coordinates, there are local minima in Lyapunov function.

On differentiating Lyapunov function we get

\[ \dot{L} = \sum_{i=1}^{N} \sum_{j=1 \atop i \neq j}^{N} K_{ij} (r_{ij} - \bar{r}_{ij}) \dot{r}_{ij} + \sum_{i=1}^{N} \dot{X}_i^T \dddot{X}_i \]

Though the Lyapunov term is positive definitive, the Lyapunov derivative is independent of the range information term.
**Lemma 1: Three Node Case**

\[ x_3 = r_{13} \cos(\theta); \quad y_3 = r_{13} \sin(\theta) \]

\[
 f_1^x = -\frac{(r_{12} - \bar{r}_{12})}{r_{12}}(x_1 - x_2) - \frac{(r_{13} - \bar{r}_{13})}{r_{13}}(x_1 - x_3)
\]

\[
 f_1^y = -\frac{(r_{12} - \bar{r}_{12})}{r_{12}}(y_1 - y_2) - \frac{(r_{13} - \bar{r}_{13})}{r_{13}}(y_1 - y_3)
\]

**Lemma 2: Four Node Case**

\[ x_1 = 0, \quad y_1 = 0; \quad x_2 = r_{12}, \quad y_2 = 0; \]

\[ x_3 = r_{13} \cos(\theta); \quad y_3 = r_{13} \sin(\theta) \]

\[ x_4 = r_{14} \cos(\theta'); \quad y_4 = r_{14} \sin(\theta') \]

\[ f_1^2 = (f_1^x)^2 + (f_1^y)^2 \]

\[ f_1^2 = a^2 + b^2 + c^2 + 2ab \cos(\theta) + 2ac \cos(\theta') + 2bc \cos(\theta'') \]

\[ r_{12} = \bar{r}_{12} \]

\[ r_{13} = \bar{r}_{13} \]
1) Initialize N=3 (number of UGS nodes in the network to start with the localization process)

2) Dynamically localize the network with N=3 using the control input defined above with range measurement information available.

3) Using the range information for the next sensor node, initialize it using the trilateration method.

4) Dynamically localize the network with the new sensor node using the control input defined above

5) Repeat steps 3-4 until all the nodes have been localized.
Relative Localization Simulation Results

Three UGS Nodes only

Four UGS Nodes

Five UGS Nodes

Six UGS Nodes

Seven UGS Nodes

Legend:
- Node 1
- Node 2
- Node 3
- Node 4
- Node 5
- Node 6
- Node 7
Modeling for Absolute Localization

The position estimates for UGS nodes with no absolute position is given as:

\[ X_{i_p} = \begin{bmatrix} x_{i_p} & y_{i_p} \end{bmatrix}^T \]
where \( x_{i_p} \) and \( y_{i_p} \) are the x-y coordinates

The position estimation dynamics are given as:

\[ \ddot{X}_{i_p} = \vec{f}_{i_p} \]
where \( \vec{f}_{i_p} = \begin{bmatrix} f_{i_p}^x & f_{i_p}^y \end{bmatrix}^T \) virtual force in the x and y directions

The position estimates for Sensor nodes with absolute position is given as:

\[ X_{i_p}^a = \begin{bmatrix} x_{i_p}^a & y_{i_p}^a \end{bmatrix}^T \]
where \( x_{i_p}^a \) and \( y_{i_p}^a \) are the x-y coordinates

The position estimation dynamics are given as:

\[ \ddot{X}_{i_p}^a = \vec{f}_{i_p}^a \]
where \( \vec{f}_{i_p}^a = \begin{bmatrix} f_{i_p}^{ax} & f_{i_p}^{ay} \end{bmatrix}^T \) virtual force in the x and y directions
Choice of Potential Field for Absolute Localization

For Absolute Localization

\[ V_p = \frac{1}{2} \sum_{p=1}^{m} K_{i_p} a \left\| e_{i_p} a \right\|^2 + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} K_{ij} (r_{ij} - \bar{r}_{ij})^2 \]

where

\[ e_{i_p} a = \left[ (x_{i_p} a - \bar{x}_{i_p} a)^2 + (y_{i_p} a - \bar{y}_{i_p} a)^2 \right]^{1/2} \]

Potential corresponding to Nodes with Absolute Position Information

Potential corresponding to Nodes with no Absolute Position Information
Choice of Potential Field for Absolute Localization: contd.

**Theorem:** Consider the position estimate dynamics $\ddot{X}_{i_p} = \ddot{f}_{i_p}$ or $\ddot{X}_{i_p}^a = \ddot{f}_{i_p}^a$ depending on the availability of absolute position information for each sensor node $i$ in the network. Let the virtual force for the $i^{th}$ sensor be given respectively as

$$f_{i_p} = \sum_{j=1}^{N} K_{i_p j} (r_{i_p j} - \bar{r}_{i_p j}) \frac{(X_{i_p} - X_j)}{\|X_{i_p} - X_j\|} - K_v \dot{X}_{i_p}$$

$$f_{i_p}^a = -K_{i_p}^a (X_{i_p}^a - \bar{X}_{i_p}^a) - \sum_{j=1}^{N} K_{i_p j} (r_{i_p j}^a - \bar{r}_{i_p j}^a) \frac{(X_{i_p}^a - X_j)}{\|X_{i_p}^a - X_j\|} - K_v^a \dot{X}_{i_p}^a$$

Then the position estimates reach steady-state values that provide optimal estimates of the actual relative localization of the nodes in the sense that $V_p$ is minimized.
Air-Ground Wireless Sensor Network Model

Assumptions:
1) UAVs have an altitude hold pilot
2) UAVs are operated in hover mode

Air-Ground sensor model consists of
1) 7 UGS node
2) 3 UAVs
Inverted configuration for UGS Network

• With only 1 Node having GPS information, the sensor network is subjected to rotation and inversion configuration.

• With 2 nodes having GPS information, the rotation uncertainty is removed but the network could still be in “upside down” configuration.

• With 3 nodes having GPS information, the upside-down uncertainty is removed and yields correctly localized network in absolute coordinates.
Solution to Upside Down Configuration

Define an error term for the 3rd node initial position given as

\[ \mathcal{E} = \left| X_{ik}^a - \bar{X}_{ik}^a \right| \]

If \( \mathcal{E} > \mathcal{E}_M \) then the network is assumed to have improper position estimates due to the estimated network being inverted.

To flip the estimated network positions upside down to the correct configuration, the orthogonal projection of all the UGS nodes already added to the network is taken on the line formed by the first 2 nodes with GPS information. The projection of the UGS nodes along the line is taken by the formula given below

\[
\begin{bmatrix}
    x^o \\
    y^o
\end{bmatrix} = \frac{1}{a^2 + b^2} \begin{bmatrix}
    b^2 - a^2 & -2ab \\
    -2ab & a^2 - b^2
\end{bmatrix} \begin{bmatrix}
    x \\
    y
\end{bmatrix} - \begin{bmatrix}
    2ac \\
    2ac
\end{bmatrix}
\]
Absolute Localization Algorithm

1) Relatively Localize the UGS nodes using Relative Localization.

2) Initialize the UAV position using trilateration method from the range measurement information available
   a) Increment i by 1 to keep a count of the number of UAVs with absolute position information
      1) if i=3 calculate $\varepsilon = \|X_{i_k}^a - \bar{X}_{i_k}^a\|
      2) Take orthogonal projection of the UGS nodes
      3) Dynamically localize the network
      a) end if

3) Dynamically localize the air-ground sensor network with the control input depending on the nodes with or with no absolute position information.

4) Repeat step 2-3 until all the 3 UAVs have been added
Absolute Localization Simulation Results

Air-Ground Sensor Network

UAV 1 with 7 UGS Nodes

UAV 1 & 2 with 7 UGS Nodes

Reflection of UGS Nodes on line formed by UAV 1 & 2

Final Configuration of Air-Ground Sensor Network
Conclusion & Future Work

• Efficient algorithms for relative and absolute localization are presented based on potential field methods, together with the mathematical analysis.

• The algorithms presented take care that the system does not fall into the local minima.

• The algorithm also takes into account the problem of the network being in an inverted configuration during absolute localization.

• Future work would be to include the UAV position dynamics
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