Observation Of The Development Of Lambda-Vortex To Hairpin Vortex Packet

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Observation of the development of \( \Lambda \)-vortex to hairpin vortex packet

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A DNS simulation is carried out to reveal the development of a \( \Lambda \)-vortex to a hairpin vortex packet. To visualize the vortices, the new omega vortex identification method is utilized. According to the observation, the counter rotation of two legs of \( \Lambda \)-vortex sweeps the low speed zone up to form the high strain rate region; the moving up of the high strain rate region is along with the generation of vortex rings; the separation of the first hairpin vortex is related to the development of the protuberance of the second vortex ring; the separation of the second and third hairpin vortices is similar with the first one, but the fourth and fifth hairpin generate in a different way from the first three hairpins.

Nomenclature

\[ \begin{align*}
M_\infty &= \text{Mach number} \\
\delta_\infty &= \text{inflow displacement thickness} \\
T_\infty &= \text{free stream temperature} \\
T_w &= \text{wall temperature} \\
in_L &= \text{height at inflow boundary} \\
out_L &= \text{height at outflow boundary} \\
L_x &= \text{length of computational domain along x direction} \\
L_y &= \text{length of computational domain along y direction} \\
x_m &= \text{distance between leading edge of flat plate and upstream boundary of computational domain} \\
x, y, z &= \text{streamwise, spanwise, normal directions} \\
V &= \text{velocity vector} \\
\omega &= \text{vorticity vector} \\
\omega_x &= \text{streamwise vorticity} \\
\omega_y &= \text{spanwise vorticity} \\
\omega_z &= \text{normal vorticity} \\
\end{align*} \]

I. Introduction

VORTEX plays a significant role in turbulence, which is called “the sinews of turbulence” \([1]\). The transition process from laminar to turbulent flow in boundary layers with the generation and development of vortices is a basic scientific problem in fluid mechanics. After over a hundred year of study, researchers have figured out the linear and weakly non-linear stage of flow transition pretty well \([2], [3]\). Many research worker studied the late non-linear transitional stage numerically and experimentally. Moin et al \([4]\) gave an explanation of the evolution of a curved vortex filament into a vortex ring in 1986. Liu et al \([5], [6]\) reported the whole process of K- and H-type transition in 1995 and 1996 and Rist \([7]\) gave a quantitative comparison of experiment and direct numerical simulation in 2002. Adrian \([8]\) gave a review about the hairpin vortex organization in boundary layer in 2007. Moin et al \([9]\) reported a new direct numerical simulation (DNS) for flow transition on a flat plate in 2009. However there are still many questions to solve in late non-linear stage of flow transition. Some misunderstandings are still popular. Even though the explanation of the evolution of a vortex filament into a vortex ring is given \([4]\), the development of \( \Lambda \)-vortex to hairpin vortex can still not be explained since the difference of vortex filament and vortex \([10]\), see Figure 1. Rigorously, the vortex

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filament which is defined as the line everywhere tangent to the local vorticity vector $\mathbf{\omega} = \nabla \times \mathbf{V}$ should be called a vorticity line, to disambiguate the misunderstanding.

Figure 1. Vortex filaments (black lines) with $\Lambda$-vortex (green surface). (Wang et al. 2016)

Some researchers believe the multiple hairpin vortices in a vortex packet is produced by auto-generation [8], however, recent researches show that the development of vortex packets is related to the high shear layer instability [10]. To study the process of $\Lambda$-vortex developing to vortex packet carefully, a detailed observation based on DNS data will be given in this paper. The case setup and code validation will be given in section II. A revisit of the new omega vortex visualization method will be given in section III. Then a detailed observation of the development of a $\Lambda$-vortex to a vortex packet will be given in section IV. And section V will summarize the conclusions.

II. Case setup and Code Validation

A. Case setup

The physical domain is displayed in Figure 2, where $x_{in}$ represents the distance between leading edge and inlet, $Lx$ and $Ly$ are the lengths of the computational domain in $x$ and $y$ directions respectively, and $Lz_{in}$ is the length of the inlet in $z$ direction. The details are listed in Table 1. The grid level is $1920 \times 128 \times 241$, representing the number of grids in streamwise ($x$), spanwise ($y$), and wall normal ($z$) directions. The grid is stretched in the normal direction and uniform in the streamwise and spanwise directions. The length of the first grid interval in the normal direction at the entrance is found to be $0.43$ in wall units ($Z^+ = 0.43$).

The Jocobian coordinate transformation is employed from physical domain to computational domain, see Figure 3(a) and the Message Passing Interface (MPI), together with domain decomposition in the $\xi$-direction, is utilized to accomplish the parallel computation, see Figure 3(b). The flow parameters, including Mach number, Reynolds number, etc. are listed in Table 2. Here, $T_w = 273.15K$ is the wall temperature.
Figure 2. Physical domain

<table>
<thead>
<tr>
<th>$x_{in}$</th>
<th>$L_x$</th>
<th>$L_y$</th>
<th>$L_z_{in}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>300.79$\delta_{in}$</td>
<td>798.03$\delta_{in}$</td>
<td>22$\delta_{in}$</td>
<td>40$\delta_{in}$</td>
</tr>
</tbody>
</table>

Table 1. Geometry parameters
B. Code validation

The DNS code − “DNSUTA” has been validated by NASA Langley and UTA researchers [11]–[13] carefully to make sure that the DNS results are correct. Since the detailed code validation has been reported by Liu and Chen [14] we only give a short description here.

1. Comparison with Log Law and grid convergence

Time and spanwise-averaged streamwise velocity profiles for various streamwise locations in two different grid levels are shown in Figure 4. The inflow velocity profiles at $x = 300.79\delta_{in}$ is a typical laminar flow velocity profile. At $x = 632.33\delta_{in}$, the mean velocity profile approaches a turbulent flow velocity profile (Log law). This comparison shows that the velocity profile from the DNS results is turbulent flow velocity profile and the grid convergence has been realized.

2. Comparison with experiment

By using $\Omega$ criterion method [15], the vortex structures shaped by the nonlinear evolution of T-S waves in the transition process are shown in Figure 5. The evolution details are studied in our previous paper [10], [16] and the formation of ring-like vortices chains is consistent with the experimental work [17], see Figure 6.
Figure 5. Evolution of vortex structure at the late-stage of transition (Where T is the period of T-S wave)
3. Comparison with Rist’s DNS data

Figure 7 shows a comparison of our DNS results with the data set provided by Rist as his personal kindness. The comparison shows both DNS have same vortex structure.

![Image](a) Our DNS  ![Image](b) Rist’s DNS data

Figure 7. Comparison of our DNS results with Rist’s DNS data.

All these verifications and validations above show that our code is correct and our DNS results are reliable.

III. Revisit of the new omega vortex identification method

Liu et al [15] proposed a new vortex identification method, $\Omega$ method, recently. The basic idea of this method is that vorticity can be decomposed into a vortical part and a non-vortical part. One conspicuous evidence of this idea is that vorticity exists in both rotating flow like vortex and flow without rotation like Blasius solution. $\Omega$ is defined as following:

$$\Omega = \frac{\|B\|_F^2}{\|A\|_F^2 + \|B\|_F^2 + \varepsilon} \quad (1)$$
where $\| \cdot \|_F$ is the Frobenius norm and $A$ is the symmetric part of velocity gradient tensor $\nabla \mathbf{V}$, $B$ is the anti-symmetric part and $\varepsilon$ is a small positive number introduced to avoid division by zero:

$$ A = \frac{1}{2} (\nabla \mathbf{V} + \nabla \mathbf{V}^T) $$

$$ B = \frac{1}{2} (\nabla \mathbf{V} - \nabla \mathbf{V}^T) $$

According to Liu et al. [15], a vortex can be identified as a connected region where $\Omega > 0.5$ and the iso-surface of $\Omega = 0.52$ can be utilized to indicate the structures of vortices.

**IV. Development of a $\Lambda$-vortex to a vortex packet**

Based on the DNS data and $\Omega$ vortex identification method, a detailed observation is carried out to figure out the process of a $\Lambda$-vortex developing to a vortex packet.

**A. $\Lambda$-vortex structure**

Figure 8 gives the structures of $\Lambda$-vortex and strain rate in different views at $t = 6.0 T$, where $T$ is T-S wave period. The green surface is the iso-surface of $\Omega = 0.52$ which indicates the structure of $\Lambda$-vortex while the yellow surface is the iso-surface of $\| A \|_F = 0.6$ which show the position of shear layer. According to Figure 8, the shear layer concentrates beyond the head of the $\Lambda$-vortex.
Figure 8. The iso-surfaces of $\Omega = 0.52$ (green) and $\|A\|_F = 0.6$ (yellow) at $t = 6.0T$, where $T$ is the period of T-S wave.

When we increase the iso-value of $\Omega$, see Figure 9, the shear layer places at the same position along streamwise direction with the iso-surface of $\Omega = 0.65$. It indicates the shear layer is high near the strong vortex. Figure 10 gives a more clear explanation. The distributions of $\omega_x$ and $u$ on the slice $x = 443$ are given in Figure 10(b) and (c). We can find a strong rotation exist at this position and the low speed zone is swept to higher position, then $\frac{\partial u}{\partial z}$ becomes greater around the position $y = 5.5, z = 2$, where the shear layer places.
Figure 9. The iso-surfaces of $\Omega = 0.65$ (green) and $\|A\|_F = 0.6$ (yellow) at $t = 6.0T$, where $T$ is the period of T-S wave.

(a) Position of the slice of $x = 443$. 
B. The ring of hairpin vortex generation

Figure 11 shows the iso-surface of $\Omega = 0.52$ and $||A||_F = 0.6$ at $t = 6.16T$, where $T$ is the period of T-S wave. From Figure 11(b), we find the first ring generates at the top of the high strain rate region. Comparing Figure 11(b) with Figure 8(c), the high strain rate region moves up. Liu et al [18] reported this phenomenon that the generation of vortex ring is due to the moving up of the shear layer. This also can be checked by Figure 12, which shows the iso-surface of $\Omega = 0.52$ and $||A||_F = 0.6$ at $t = 6.30T$. Comparing Figure 12 with Figure 11(b), the second high strain rate region moves upper at $t = 6.30T$. Then the second ring generates following as a result.
Figure 11. The iso-surface of $\Omega = 0.52$ and $\|A\|_F = 0.6$ at $t = 6.16T$, where $T$ is the period of T-S wave.

Figure 12. The iso-surface of $\Omega = 0.52$ and $\|A\|_F = 0.6$ at $t = 6.30T$, where $T$ is the period of T-S wave.

C. The first three hairpins separating from $\Lambda$-vortex

Note that in Figure 12, there is a protuberance at the “mandible” of the second ring. This protuberance grows along lower front direction to enfold the legs of the first vortex ring. It is more clear in Figure 13. Figure 13 gives the vortex structures at $t = 6.43T$ in global, X-Z and X-Y view. The red arrows indicate the direction of the protuberance developing.
Figure 13. The iso-surface of $\Omega = 0.52$ at $t = 6.43T$, where $T$ is the period of T-S wave.

Figure 14 shows the vortex structures at $t = 6.43T$ in X-Z view, X-Y top view and bottom view. The protuberance extends to the bottom and is parallel to the leg of the first hairpin vortex, see Figure 14(a). And the first hairpin vortex has been separated from the $\Lambda$-vortex, see Figure 14(c).
The distribution of $\omega_x$ on the slice of $x = 475$ with stream traces (black lines) at $t=6.60T$ is shown in Figure 15. We can find the protuberance and the leg of the first hairpin are counter-rotating, see Figure 15(b). This is the possible reason that the first hairpin vortex separate from the $\Lambda$-vortex. Also from Figure 15(a), we can find the separations of the first three hairpin vortex from $\Lambda$-vortex have the similar process, that is, the separations are all with the extension of the protuberance of latter one.
Figure 15. The distribution of $\omega_x$ on the slice of $x = 475$ with stream traces (black lines) at $t = 6.90T$, where $T$ is the period of T-S wave.

D. The fourth and fifth hairpin vortices generation

The process of the generation of the fourth and fifth hairpin vortices is different from the first three hairpins, see Figure 16. The details will be developed in the complete paper.
V. Conclusion

A DNS simulation is carried out to reveal the development of a $\Lambda$-vortex to a hairpin vortex packet. The new omega vortex identification method is utilized to visualize the vortices. According to the observation, the high strain rate region on the top of $\Lambda$-vortex is caused by the sweep due to the counter rotation of two legs of $\Lambda$-vortex; the generation of vortex rings is related to the moving up of high strain rate region; the separation of the first hairpin vortex is related to the development of the protuberance of the second vortex ring; the separation of the second and third hairpin vortices is similar with the first one, but the fourth and fifth hairpin generate in a different way from the first three hairpins, which will be studies in future and concluded in the complete paper.

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