Basic Concepts and Distinctions

Logic

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Terms in boldface type are defined in this handout.

1. **Logic** is the study of relations between **propositions** (or propositional forms). There are nine important relations, with the first listed—logical implication—being of special importance:

   a. **Logical implication** (also known as “entailment”).
   b. Logical equivalence.
   c. Contradictoriness.
   d. Contrariety.
   e. Subcontrariety.
   f. Subalternation.
   g. Independence.
   h. Consistency.
   i. Inconsistency.

See the handout entitled “Logical Relations Between Propositions” for details. We will study these relations—and their relations to one another!—in due course.

2. **Propositions** are (among other things) bearers of truth value. In this course, we shall assume that there are two (and only two) truth values: **true** and **false**. Here are two Laws of Thought:

   a. Every proposition is either true or false, i.e., the truth values ‘true’ and ‘false’ are **jointly exhaustive** (this is known as the Law of Excluded Middle or, more precisely, the Law of Bivalence); and
   b. No proposition is both true and false, i.e., the truth values ‘true’ and ‘false’ are **mutually exclusive** (this is known as the Law of Noncontradiction).

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1 “Albuquerque is west of the Mississippi River” is (or rather, express) a proposition, namely, that Albuquerque is west of the Mississippi River. ‘x is west of y’ is a propositional form.
2 In what is called “many-valued logic,” there are more than two truth values.
3 Strictly speaking, these are *entailments* of the Laws of Thought, not Laws of Thought themselves.
4 While every proposition is either true or false and no proposition is both true and false, it doesn’t follow that we always know which truth value a given proposition has. Take the proposition that Abraham Lincoln thought about his son Robert five seconds before he (Abraham) was shot. This proposition is either true or false and it is not both true and false, but we will probably never know its truth value. (Query: What would constitute evidence for its truth? What would constitute evidence for
Sentences, unlike propositions, are linguistic entities, which means that they are in particular languages, such as English, German, Swahili, or Latin. Propositions, which are in no particular language, are what indicative (declarative) sentences express, assert, or signify. Two different indicative (declarative) sentences (e.g., “John loves Mary” and “Mary is loved by John” [both of which are in English], or “It is raining” and “Il pleut” [the former of which is in English, the latter of which is in French]) can express, assert, or signify the same proposition. This property of sentences is called synonymy. A given indicative (declarative) sentence (e.g., “Jones is happy”) can express, assert, or signify different propositions, depending on such things as (a) when it is uttered (Jones may be happy at one time but not at another) and (b) what its terms (e.g., “Jones”) refer to (Adam Jones may be happy while Andrew Jones is not). This property of sentences is called ambiguity.

3. Inference (or reasoning) is the psychological (mental) process by which one proposition (known as the conclusion) is derived from, or arrived at on the basis of, one or more other propositions (known as the premises). The word “inference” may be used to refer either to the process (i.e., the act of inferring) or to the product of the process (i.e., the proposition inferred). For example, suppose I infer from the fact that there are dark clouds approaching that it will rain soon. I have performed an act of inference (or reasoning). My inference (conclusion) is that it will rain soon. Suppose I infer from the fact that Donald J. Trump is the 45th president of the United States that there have been 44 other presidents. I have performed an act of inference (or reasoning). My inference (conclusion) is that there have been 44 presidents other than Donald J. Trump.

4. Every inference (using the word now to refer to the process rather than to the product of the process) can be expressed as, or transformed into, an argument, and every argument is the expression or statement of an inference. The purpose of an argument is to persuade or convince someone to believe something (the conclusion) by providing reasons, evidence, or grounds for it. (These reasons, evidence, or grounds constitute the premises.) Thus, an argument consists of two or more propositions, one of which (the conclusion) is claimed (by the arguer) to follow from the other or others (the premises). The act or process of arguing is known as argumentation. Argumentation is interpersonal or public; inference is intrapersonal or private.

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5 Sentence is to proposition as numeral is to number.
6 Some people refer to it as “synonymity.”
7 Among those who seek to persuade or convince are marketers, politicians, preachers, and moralizers, but also family members, friends, colleagues, fellow students, teachers, teammates, neighbors, and compatriots.
8 I will focus on arguments rather than inferences in the remainder of this handout, but keep in mind that much of what I say about arguments can also be said about inferences.
5. Every argument, by definition (see immediately above), involves a \textit{claim} (by the arguer) that its conclusion follows from its premises. Some claims are stronger than others:

a. If one’s claim is that the conclusion \textit{cannot} be false, given the truth of the premises, then one’s argument is \textbf{deductive}. (Another way to put this is that, in a deductive argument, the arguer claims that the premises \textit{logically imply}, or \textit{entail}, the conclusion.)

b. If one’s claim is merely that the conclusion is \textit{unlikely} to be false, given the truth of the premises, then one’s argument is \textbf{inductive}. (The arguer in this case does \textit{not} claim that the premises logically imply, or entail, the conclusion, only that they provide support for the truth of the conclusion.)

Deduction is to necessity as induction is to probability. What follows is a deductive argument (for almost certainly the arguer would claim that the premises logically imply the conclusion):

\begin{itemize}
  \item All mammals are animals.
  \item All dogs are mammals.
  \item Therefore,
  \item All dogs are animals.\textsuperscript{9}
\end{itemize}

Here, by way of contrast, is an inductive argument (for almost certainly the arguer would \textit{not} claim that the premises logically imply the conclusion):

\begin{itemize}
  \item Most professors are atheists.
  \item Jones is a professor.
  \item Therefore,
  \item Jones is an atheist.\textsuperscript{10}
\end{itemize}

The focus of this course is deduction. Our concern will be arguments in which the arguer claims that the premises logically imply, or entail, the conclusion.

6. \textbf{Logical implication} (also known as “entailment”) is a relation between \textbf{propositions} (or propositional forms). It is not to be confused with \textit{material} implication (which we will discuss later in the course), so the adjectives “logical” and “material” are necessary (unless the context makes it clear which type of implication is intended). One proposition logically implies (“entails”) a second proposition

\begin{itemize}
  \item \textsuperscript{9} Another example: “Every mammal has a heart; all horses are mammals; therefore, every horse has a heart.” (From Wesley C. Salmon, \textit{Logic}, 3d ed. [Englewood Cliffs, NJ: Prentice-Hall, 1984], 14.)
  \item \textsuperscript{10} Another example: “Every horse that has ever been observed has had a heart; therefore, every horse has a heart.” (From Salmon, \textit{Logic}, 3d ed., 14.)
\end{itemize}
when it is logically impossible for the first to be \textbf{true} while the second is \textbf{false}. Example: ‘x is (exactly) six feet tall’ logically implies ‘x is at least six feet tall’, for it is logically impossible for x to be (exactly) six feet tall without x being at least six feet tall. Note that the second of these propositions does not logically imply the first, for x may be at least six feet tall without x being (exactly) six feet tall. Some pairs of propositions logically imply each other—for example, ‘x is (exactly) six feet tall’ and ‘x is (exactly) 72 inches tall’. Henceforth, “impossible” will mean “logically impossible”; “possible” will mean “logically possible”; and “necessary” will mean “logically necessary.”

7. Every deductive argument (and therefore every inference expressed by a deductive argument) is either \textbf{valid} or \textbf{invalid}, depending on whether the claim of logical implication made by the arguer is true. If the premises do (in fact) logically imply the conclusion, then the arguer’s claim is true and the argument is valid. If the premises do \textit{not} (in fact) logically imply the conclusion, then the arguer’s claim is false and the argument is invalid. Note that whether an argument is deductive or inductive depends on the \textit{strength} of the claim being made by the arguer. Whether a deductive argument is valid or invalid depends on the \textit{truth} of the claim being made by the arguer. It is important not to conflate these points.

Strictly speaking, only \textit{deductive} arguments are valid or invalid. We may, if we like, say that all inductive arguments are invalid, but this is misleading, for it suggests that there is something wrong with inductive arguments. There is nothing wrong with inductive arguments.\footnote{Some inductive arguments are cogent, in the sense that (a) they have true premises and (b) their premises make their conclusions probable. Example: “Most dogs have a tail of at least one inch in length (true); Shelbie is a dog (true); therefore, Shelbie has a tail of at least one inch in length.” Perhaps it’s best to say that the terms “valid” and “invalid” don’t \textit{apply} to inductive arguments, i.e., that inductive arguments are neither valid nor invalid. They belong in category 3 of the following taxonomy:}$^{13}$

\begin{itemize}
  \item \textbf{valid} (i.e., truth-preserving) deductive argument,
  \item \textbf{invalid} (non-truth-preserving) deductive argument,
\end{itemize}

\footnote{The reason for this stipulation is that there are different types of possibility (and hence different types of impossibility and necessity). A given act, for example, may be \textit{psychologically} possible, impossible, or necessary. A given event may be \textit{physically} possible, impossible, or necessary. Since our concern in this course is \textit{logical} possibility, impossibility, and necessity, rather than some other kind, there is no need to keep using the adjective “logical.”}

\footnote{“Without some type of inductive reasoning, we would have no grounds for predicting that night will continue to follow day, that the seasons will continue to occur in their customary sequence, or that sugar will continue to taste sweet. All such knowledge of the future, and much else as well, depends upon the power of inductive arguments to support conclusions that \textit{go} beyond the data presented in their premises” (Salmon, \textit{Logic}, 3d ed., 18).}

\footnote{A taxonomy is a classification scheme. The taxonomy in the text is both jointly exhaustive and mutually exclusive (because of how the categories are defined). To say that it is jointly exhaustive is to say that every argument goes in at least one of its three categories. To say that it is mutually exclusive is to say that no argument goes in more than one of its three categories. It follows that every argument goes in \textit{exactly} one of its three categories. Every argument, in other words, is either (i) a valid (i.e., truth-preserving) deductive argument, (ii) an invalid (non-truth-preserving) deductive argument,
### Arguments

<table>
<thead>
<tr>
<th>Valid</th>
<th>Not Valid</th>
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<tbody>
<tr>
<td></td>
<td>Invalid</td>
</tr>
<tr>
<td>1</td>
<td>Not Invalid</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
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</table>

8. A **valid argument** is a deductive argument that has the following property: it is impossible for its premises to be true while its conclusion is false. (If it is **possible** for a given deductive argument’s premises to be true while its conclusion is false, then it is invalid.) Valid arguments are truth-preserving, in the following sense:

- a. If the *premises* of a valid argument are *true*, then its conclusion must be true.
- b. If the *conclusion* of a valid argument is *false*, then at least one of its premises must be false.

Note that there can be a valid argument with one or more false premises as well as an invalid argument with true premises. An example of the former is:

Texas is east of the Mississippi River.  
Therefore,  
Something is east of the Mississippi River.

An example of the latter is:

Something is east of the Mississippi River.  
Therefore,  
Texas is east of the Mississippi River.

Note also that, while all valid arguments are deductive, not all deductive arguments are valid. What makes an argument *deductive* (as we saw earlier) is the **strength** of the claim being made by the arguer. What makes a deductive argument *valid* (we are now seeing) is the **truth** of the claim being made by the arguer. As I said earlier, this course is concerned exclusively with deduction. If you wish to study induction, you should take Fundamentals of Reasoning (PHIL 1301).  

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or (3) an argument that is neither valid nor invalid. Category 3 could, of course, be subdivided, for it contains all inductive arguments, some of which are cogent (such as the one in the text) and some of which are not cogent (e.g., “Eighty-five percent of the readers of the *New York Times* oppose capital punishment; therefore, 85% of Americans oppose capital punishment”).

14 The two courses may be taken in any order; neither, in other words, is a prerequisite for the other. Some people disparage Fundamentals of Reasoning (which at some universities is known as Critical Thinking or Informal Logic) as “baby logic.” That’s like saying that abnormal psychology is
9. A **sound argument** is a **valid argument** the premises of which are true.\(^6\) It follows from this definition that:

a. All sound arguments have true conclusions. (Note the difference between *x following from* a definition and *x being, or being part of,* a definition.)\(^6\)

b. Any argument that is invalid is unsound.

c. Any argument that has a false premise (even one) is unsound.

A given argument may be invalid and have a false premise. Such an argument has two defects, one of them formal (invalidity) and the other material (a false premise). Here is an example of a doubly defective argument:

Keith’s automobile is green.
Therefore,
Everything is green.

As a matter of fact, Keith’s automobile is not green; but even if it were, it would not follow that everything is green.\(^7\) (This is known as a *non sequitur*—i.e., a non-follower.) Another example is provided by the philosopher Judith Jarvis Thomson, in her 1971 essay, “A Defense of Abortion”:

Fetuses have a right to life.
Abortion kills fetuses.
Therefore,
Abortion is wrong.

Thomson says that the first premise of this argument is false, but adds that even if it (together with the second premise) were true, the conclusion would not follow. This is known as assuming something for the sake of argument (or playing devil’s

\(^6\) We might put this as an equation: \(S = V + T\).

\(^6\) Suppose I put the following statement on an examination: “By definition, all sound arguments have true conclusions.” The statement is false. While it’s *true* that all sound arguments have true conclusions, it’s not true *by definition.* The definition of “sound argument” makes no reference to the conclusion of the argument, much less to the conclusion being true. The definition of “sound argument” makes reference to two things: (i) validity; and (ii) the truth of the premises. When you put these two components of the definition together, it follows that all sound arguments have true conclusions.

\(^7\) If one’s aim is to criticize (i.e., find fault with) another person’s argument, one should identify *all* of its defects, formal as well as material. Conversely, if one is making an argument, one should ensure not only that one’s conclusion follows from one’s premises (i.e., that it has a truth-preserving form), but that one’s premises are (in fact) true.
Here, in the form of a taxonomy (of deductive arguments), is a summary of this section:

<table>
<thead>
<tr>
<th>All of the Premises Are True</th>
<th>Not All of the Premises Are True (i.e., at Least One of the Premises Is False)</th>
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<tbody>
<tr>
<td>Valid</td>
<td>Sound</td>
</tr>
<tr>
<td>Invalid</td>
<td>Unsound</td>
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</table>

See the handouts entitled “Argument Analysis” and “Understanding Validity” for details.

Validity and soundness are all or nothing, not matters of degree. (Figuratively speaking, they are digital, not analog.) A given deductive argument is either valid or invalid, sound or unsound. It makes no sense to say, of a deductive argument, that it is “almost valid” or “almost sound,” or that one deductive argument is “valider” (more valid) or “sounder” (more sound) than another. If a given deductive argument has 1,000,000 premises and 999,999 of them are true (the remaining premise being false), then the argument is unsound, just as it would be if all 1,000,000 premises were false. Here, then, is our final taxonomy of arguments:

<table>
<thead>
<tr>
<th>Arguments</th>
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<tbody>
<tr>
<td>Deductive</td>
</tr>
<tr>
<td>Valid</td>
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<tr>
<td>Sound</td>
</tr>
</tbody>
</table>

The taxonomy is jointly exhaustive in that every argument goes in at least one of its four categories. Every argument, in other words, is either (1) sound (i.e., a valid deductive argument with true premises), (2) unsound (i.e., a valid deductive argument with at least one false premise), (3) invalid (i.e., a deductive argument that is not truth-preserving), or (4) inductive (i.e., an argument that is not deductive, but which may well be cogent). The taxonomy is mutually exclusive in that no argument goes in more than one of its four categories (i.e., every argument goes in at most one of its four categories). It follows that every argument goes in exactly one of the taxonomy’s four categories.

Validity (a property of some, but not all, deductive arguments) is valuable (i.e., worthy of being valued) for the sake of what it preserves, namely, truth.
It is not valuable for its own sake.\(^8\) Truth (i.e., true belief) is valuable because it is essential to knowledge. (One can't \textit{know} falsehoods, though one can \textit{believe} falsehoods.) Knowledge is valuable because it is a component of the good life. As the Greek philosopher Socrates put it (according to his disciple Plato), “The unexamined life is not worth living.”\(^9\)

12. Specialists in \textit{logic} are known as logicians.\(^10\) Logicians, \textit{as such}, have no expertise in determining which propositions are true and which false—unless the propositions in question are true or false \textit{simply by virtue of their form} (such as “God exists or God does not exist,” which is \textit{true} by virtue of its form, and “God exists and God does not exist,” which is \textit{false} by virtue of its form).\(^21\) Logicians are, however, expert in determining which arguments (argument forms) are valid, for logic, as we saw at the outset, is the study of relations between propositions, and validity is simply the relation of logical implication (entailment) applied to the premises and conclusions of arguments.\(^22\)

\textbf{APPENDIX: DEFINITIONS OF “LOGIC”}


\(^{18}\) Another way to put this is that validity is extrinsically or instrumentally valuable, not intrinsically valuable. Validity is a means to an end, not an end in itself. In this respect, validity is like money and unlike, say, friendship, knowledge, pleasure, or beauty, each of which is (arguably) intrinsically valuable.

\(^{19}\) See Plato’s \textit{Apology}, which every educated person should read. It is available online.

\(^{20}\) Specialists in magic are known as magicians—which is not to say that logic has anything to do with magic!

\(^{21}\) Propositions that are true by virtue of their form are called “tautologies.” Propositions that are false by virtue of their form are called “self-contradictions.” We will have more to say about tautologies and self-contradictions in due course.

\(^{22}\) People who are not expert in a given field should defer to those who are. If you are not a logician, then you should defer to logicians on matters that are within the scope of their expertise, just as, if you are not a biologist, then you should defer to biologists on matters that are within the scope of their expertise. It is important to understand that expertise does not transfer from one field to another, so the fact that a particular individual is expert in field X does not mean that he or she is expert in field Y. Some people, unfortunately, are expert in nothing. (A person of this sort is said to be a “jack of all trades, master of none.”) Some people, fortunately, are expert in more than one field. I have a friend (Robert “Bob” Schopp, who teaches at the University of Nebraska) who has three advanced degrees: a Ph.D. in psychology, a Ph.D. in philosophy, and a J.D. (law). He works at the intersection of these fields—on topics such as insanity and automatism (both of which are criminal defenses). People who are expert in more than one field—especially people, such as Bob, who are expert in \textit{multiple} fields—are known as “polymaths.” Aristotle (384–322 BCE), Gottfried Wilhelm Leibniz (1646–1716), and Immanuel Kant (1724–1804) were polymaths.
<table>
<thead>
<tr>
<th>Statement</th>
<th>Source</th>
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<tbody>
<tr>
<td>“Loosely speaking, logic is the process of correct reasoning, and something is logical when it makes sense. Philosophers often reserve this word for things having to do with various theories of correct reasoning.”</td>
<td>Robert M. Martin, <em>The Philosopher’s Dictionary</em>, 3d ed. (Orchard Park, NY: Broadview Press, 2002), 182.</td>
</tr>
<tr>
<td>“The scope of the term ‘logic’ has varied widely from writer to writer through the centuries. But these varying scopes seem all to enclose a common part: the logic which is commonly described, vaguely, as the science of necessary inference.”</td>
<td>Willard Van Orman Quine, <em>Elementary Logic</em>, rev. ed. (Cambridge: Harvard University Press, 1980), 1.</td>
</tr>
<tr>
<td>“Logic is the study of principles of reasoning. It is concerned not with how people actually reason, but rather with how people ought to reason if they wish to ensure the truth of their results.”</td>
<td>Warren Goldfarb, <em>Deductive Logic</em> (Indianapolis: Hackett Publishing Company, 2003), xiii.</td>
</tr>
<tr>
<td>“Logic deals with arguments and inferences. One of its main purposes is to provide methods for distinguishing those that are logically correct from those that are not.”</td>
<td>Wesley C. Salmon, <em>Logic</em>, 3d ed. (Englewood Cliffs, NJ: Prentice-Hall, 1984), 1.</td>
</tr>
<tr>
<td>“Logical inference leads from premises—statements assumed or believed for whatever reason—to conclusions which can be shown on purely logical grounds to be true if the premises are true. Techniques to this end are a primary business of logic. . . .”</td>
<td>W. V. Quine, <em>Methods of Logic</em>, 4th ed. (Cambridge: Harvard University Press, 1982), 53 (italics in original).</td>
</tr>
<tr>
<td>“Logic is the autonomous science of the objective though formal conditions</td>
<td>Morris R. Cohen and Ernest Nagel, <em>An Introduction to Logic</em> (New York</td>
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</table>
Logic is "the science of valid inference."

Logic is "the science of the weight of evidence in all fields."

"Logic may be said to be concerned with the question of the adequacy or probative value of different kinds of evidence. Traditionally, however, it has devoted itself in the main to the study of what constitutes proof, that is, complete or conclusive evidence."

"Logic as a distinctive science is concerned ... with the relation of implication between propositions. Thus the specific task of logic is the study of the conditions under which one proposition necessarily follows and may therefore be deduced from one or more others, regardless of whether the latter are in fact true."

"It is the object of logical study to consider more detailed rules for distinguishing valid from invalid forms of argument."

"Logic may ... be ... defined as the science of implication, or of valid inference (based on such implication)."

"The essential purpose of logic is attained if we can analyze the various forms of inference and arrive at a systematic way of discriminating the valid from the invalid forms."

"Logic may be conceived as ruling out and Burlingame: Harcourt, Brace & World, 1962 (first published in 1934 as Book I of An Introduction to Logic and Scientific Method), viii, viii, viii, 5, 8, 12, 13, 20, 21, 110, 182, 185-6 (italics in original).
what is absolutely impossible, and thus
determining the field of what in the ab-
sence of empirical knowledge is ab-
stractly possible.”

“[T]he fundamental task of logic is the
study of those objective relations be-
tween propositions which condition
the validity of the inference by which
we pass from premises to conclusions.”

“Logic studies the relations between
sets of propositions in virtue of which
some limitation is placed upon the pos-
sible truth or falsity of one set by the
possible truth or falsity of another set.”

“Logic may be regarded as the study
of the most general, the most pervasive
characters of both whatever is and
whatever may be.”

“Logic may be defined as the organized
body of knowledge, or science, that
evaluates arguments.”

“Logic is the study of reasoning.”

| “The subject matter of symbolic logic
  is merely logic—the principles which
govern the validity of inference.” |
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| Clarence Irving Lewis and Cooper
  Harold Langford, *Symbolic Logic*,
  2d ed. (New York: Dover Publications,
  1959), 3 (italics in original). |

| “It is quite common for people to con-
  centrate on the individual statements
  in an argument and investigate whether
  they are true or false. Since people
  want to know things, the actual truth
  or falsity of statements is important;
  but it is not the only important ques-
  tion. Equally important is the question
  ‘Assuming the premises are true, do
  they support the conclusion?’ This
  question offers a glimpse of the role of
  logic, which is the study of reasoning,
  and the evaluation of arguments.” |
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| Stan Baronett, *Logic*, 3d ed. (New
  York: Oxford University Press,
  2016), 3. |

<table>
<thead>
<tr>
<th>“Logic is the study of reasoning.”</th>
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<tbody>
<tr>
<td>Daniel Bonevac, <em>Simple Logic</em> (New</td>
</tr>
<tr>
<td>“Logic may be broadly defined as the study of methods for determining whether or not a conclusion has been correctly drawn from a set of assumptions.”</td>
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|“The province of logic must be restricted to that portion of our knowledge which consists of inferences from truths previously known; whether those antecedent data be general propositions, or particular observations and perceptions. Logic is not the science of Belief, but the science of Proof, or Evidence. In so far as belief professes to be founded on proof, the office of logic is to supply a test for ascertaining whether or not the belief is well grounded. With the claims which any| John Stuart Mill, *A System of Logic Ratiocinative and Inductive: Being a Connected View of the Principles of Evidence and the Methods of Scientific Investigation*, Collected Works of John Stuart Mill, vol. VII (Toronto: University of Toronto Press, 1974 [first published in 1843]), 9. |
A proposition has to believe on the evidence of consciousness, that is, without evidence in the proper sense of the word, logic has nothing to do."

"Logical implication is the well-defined core of implication, and the techniques governing it are the central business of logic."

"Implication is what makes our system of beliefs cohere. If we see that a sentence is implied by sentences that we believe true, we are obliged to believe it true as well, or else change our minds about one or another of the sentences that jointly implied it. If we see that the negation of some sentence is implied by sentences that we believe true, we are obliged to disbelieve that sentence or else change our minds about one of the others. Implication is thus the very texture of our web of belief, and logic is the theory that traces it."

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