Exercise Set 2 (Basic Concepts and Distinctions)
Logic
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For each of the following exercises, you are given a piece of information about an argument (call the argument “X”). Based on (1) this information and (2) your knowledge of the relevant concepts, determine what else, if anything, you know about the argument, and why. Be thorough. (In other words, tell me everything you know about the argument.)

1. X has a false conclusion.
2. X has a true conclusion.
3. X has at least one false premise.
4. X has at least one true premise.
5. X has true premises (i.e., all of X’s premises are true).
6. X has false premises (i.e., all of X’s premises are false).
7. X is sound.
8. X is unsound.
9. X is valid.
10. X is invalid.
11. X is valid and has true premises.
12. X is valid and has false premises.
13. X is valid and has a false conclusion.
14. X is valid and has a true conclusion.
15. X is invalid and has true premises.
16. X is invalid and has at least one false premise.
17. X has true premises and a false conclusion.

Answers:

The key to doing these exercises is understanding the concept of validity. By definition, there cannot be a valid argument with true premises and a false conclusion. No argument, in other words, has all the following properties (features, characteristics):

- is valid
- has true premises
- has a false conclusion
If you know that a particular argument has *two* of these properties, you know that it lacks the third. Here are the possibilities:

- If a *valid* argument has *true premises*, then it cannot have a *false conclusion* (i.e., it must have a true conclusion).
- If a *valid* argument has a *false conclusion*, then it cannot have *true premises* (i.e., it must have at least one false premise).
- If an argument has *true premises* and a *false conclusion*, then it cannot be *valid* (i.e., it must be invalid).

If you know that a particular argument has *one* of these properties, you know that it lacks at least one of the other two (and possibly both). Here are the possibilities:

- If an argument is *valid*, then *either* it has a *false premise* or it has a *true conclusion* (or both).
- If an argument has *true premises*, then *either* it is *invalid* or it has a *true conclusion* (or both).
- If an argument has a *false conclusion*, then *either* it is *invalid* or it has a *false premise* (or both).

1. **X has a false conclusion.** By definition, no valid argument has both true premises and a false conclusion, so, if X’s conclusion is false, then *either* X is invalid or X has a false premise (or both). Also, since all sound arguments have true conclusions and X has a false conclusion, X is unsound.

2. **X has a true conclusion.** All sound arguments have true conclusions, but not all arguments with true conclusions are sound. So it is unknown whether X is sound. Both valid and invalid arguments can have true conclusions, so it is unknown whether X is valid. Nothing is known (can be inferred) about X’s premises.

3. **X has at least one false premise.** A sound argument, by definition, has true premises, so X is unsound. Nothing further is known.

4. **X has at least one true premise.** If the implication is that X has at least one false premise, then X is unsound, since sound arguments, by definition, have true premises. If this is not being implied, then it is unknown whether X is sound. Nothing further is known.

5. **X has true premises (i.e., all of X’s premises are true).** Either X is invalid
or X has a true conclusion (or both). (See above: If an argument has true premises, then either it is invalid or it has a true conclusion (or both)). Nothing further is known.

6. X has false premises (i.e., all of X's premises are false). X is unsound (since a sound argument, by definition, has true premises).

7. X is sound. X is valid and has true premises (by definition). Also, X has a true conclusion (by inference).

8. X is unsound. Either X is invalid or X has at least one false premise (or both).

9. X is valid. Either X has a false premise or X has a true conclusion (or both). (See above: If an argument is valid, then either it has a false premise or it has a true conclusion (or both)). All sound arguments are valid, but not all valid arguments are sound, so it is unknown whether X is sound.

10. X is invalid. X has at least one false premise. X is unsound (since a sound argument, by definition, is valid).

11. X is valid and has true premises. X has a true conclusion. (See above: If a valid argument has true premises, then it cannot have a false conclusion [i.e., it must have a true conclusion]). Also, X is sound (by definition).

12. X is valid and has false premises. X is unsound (since a sound argument, by definition, has true premises). Nothing is known about X's conclusion.

13. X is valid and has a false conclusion. X has at least one false premise. (See above: If a valid argument has a false conclusion, then it cannot have true premises [i.e., it must have at least one false premise]). Also, X is unsound (since all sound arguments have true conclusions).

14. X is valid and has a true conclusion. Nothing is known about X's premises. There can be a valid argument with true premises and a true conclusion just as there can be a valid argument with at least one false premise and a true conclusion. Therefore, it is unknown whether X is sound.

15. X is invalid and has true premises. X is unsound (since a sound argument, by definition, is valid). Nothing is known about X’s conclusion.

16. X is invalid and has at least one false premise. X is unsound (since a sound argument, by definition, is valid). Nothing is known about X’s conclusion.
17. **X has true premises and a false conclusion.** X is invalid. (See above: If an argument has **true premises** and a **false conclusion**, then it cannot be **valid** [i.e., it must be invalid]). Also, X is unsound (since a sound argument, by definition, is valid).