addition (Add). In propositional logic, a rule of inference (i.e., an elementary valid argument form) in which (1) the conclusion is a disjunction and (2) the premise is the first disjunct of the conclusion. Formally, 'p'; therefore, 'p ∨ q'. See (1) disjunction, (2) elementary valid argument form, (3) inference, rules of, and (4) propositional logic.

affirmative proposition. In categorical logic, a proposition that asserts that one class is included (i.e., contained) in another, either totally or partially. A proposition that affirms class membership. If the claim is one of totality, the proposition is universal (‘All S are P’). If the claim is one of partiality, the proposition is particular (‘Some S are P’). See (1) “A” proposition, (2) “I” proposition, and (3) negative proposition.

affirming the consequent (AC), fallacy of (a.k.a. asserting the consequent). In propositional logic, an invalid syllogism (i.e., a formal fallacy) in which the first premise is a conditional, the second premise the consequent of that conditional, and the conclusion the antecedent of that conditional. The name derives from the fact that the second premise affirms the consequent of the first premise. Formally, ‘p ⊃ q’; ‘q’; therefore, ‘p’. See (1) denying the antecedent (DA), fallacy of and (2) Modus Ponens.

affirma. Latin for “I affirm.” In categorical logic, the letter names “A” and “I” come from the first two vowels of the word “affirma.” The “A” proposition is universal affirmative; the “I” proposition is particular affirmative. See (1) “A” proposition, (2) nego, and (3) “I” proposition.

“all.” The universal affirmative quantifier, as in ‘All S are P’. See (1) “no” and (2) “some.”

ambiguity (a.k.a. equivocation). A property of (some) linguistic entities, such as words and sentences. A term (word or sentence) is ambiguous in a given context when it has two or more distinct meanings and the context does not make clear which meaning is intended by the utterer. Examples: “bank,” “right,” “duty,” “material implication.” See (1) sentence, (2) synonymy, and (3) vagueness.

antecedent (a.k.a. protasis). In propositional logic, the part of a conditional
that follows the word “if,” or, in the case of a symbolized expression, precedes the horseshoe. Example: In the conditional “If this is an even-numbered year, then there are Congressional elections this year,” the antecedent is “this is an even-numbered year.” In the symbolized expression ‘\( q \supset p \),’ the antecedent is ‘\( q \).’ See (1) conditional and (2) consequent.

**antilogism.** An inconsistent triad of propositions. A triad of propositions such that the truth of any two of them logically implies the falsity of the third. A valid syllogism is a syllogism whose premises, taken with the contradictory of the conclusion, constitute an antilogism. Example:

<table>
<thead>
<tr>
<th>Valid Syllogism</th>
<th>Antilogism</th>
</tr>
</thead>
<tbody>
<tr>
<td>All men are mortal.</td>
<td>All men are mortal.</td>
</tr>
<tr>
<td>Socrates is a man.</td>
<td>Socrates is a man.</td>
</tr>
<tr>
<td>( \therefore ) Socrates is mortal.</td>
<td>Socrates is not mortal.</td>
</tr>
</tbody>
</table>

**“A” proposition.** In categorical logic, a universal affirmative standard-form categorical proposition: ‘All S are P’. See (1) “E” proposition, (2) “I” proposition, (3) “O” proposition, and (4) standard-form categorical proposition (SFCP).

**“all.”** The universal affirmative quantifier, as in ‘All S are P’. See (1) “no” and (2) “some.”

**argument.** The expression of an inference. Any group of (two or more) propositions of which one, the conclusion, is claimed (by the arguer) to follow from the other or others, the premise(s). The premise or premises are regarded as providing support, grounds, reasons, or evidence for the truth of the conclusion. Every argument has at least one premise and exactly one conclusion, though there are chain arguments that consist of two or more arguments linked together, with the conclusion of one serving as a premise of another. See (1) argument form, (2) conclusion, (3) chain argument, (4) inference, and (5) premise.

**argument form.** Any array of symbols containing propositional variables (‘p’, ‘q’, ‘r’, ‘s’, and so forth) but no propositions, such that when propositions are substituted for the propositional variables—the same proposition being substituted for the same propositional variable throughout—the result is an argument. See (1) argument and (2) substitution instance.

**argumentation.** The act, process, practice, or institution of arguing, or producing an argument. The aim of argumentation is to persuade or convince someone to believe or do something. See argument.
Aristotelian interpretation of standard-form categorical propositions.
See (1) Aristotle and (2) square of opposition.

Aristotle, Greek Aristoteles (born 384 BCE, Stagira, Chalcidice, Greece—died 322, Chalcis, Euboea). Ancient Greek philosopher and scientist, one of the greatest intellectual figures of Western history. He was the author of a philosophical and scientific system that became the framework and vehicle for both Christian Scholasticism and medieval Islamic philosophy. Even after the intellectual revolutions of the Renaissance, the Reformation, and the Enlightenment, Aristotelian concepts remained embedded in Western thinking (from Encyclopedia Britannica online). See categorical logic.

artificial symbolic language. A language created by logicians to avoid some of the problems that inhere in natural language, such as vagueness, ambiguity (substantive or structural), misleading idioms, emotive meaning, and confusing metaphorical style. The special symbols of modern logic (propositional and predicate) help us to exhibit with greater clarity the logical structures of propositions and arguments. See natural language.

association (Assoc). In propositional logic, two replacement rules. The first says that three disjuncts may be reassociated with one another (i.e., that parentheses may be relocated). Formally, ‘p ∨ (q ∨ r)’ :: ‘(p ∨ q) ∨ r’. The second says that three conjuncts may be reassociated with one another. Formally, ‘p • (q • r)’ :: ‘(p • q) • r’.

asyllogistic inference. In predicate logic, an inference (argument) that involves propositions with more complicated internal structures than either standard-form categorical propositions (“A,” “E,” “I,” or “O”) or singular propositions. For example, “Hotels are both expensive and depressing; some hotels are shabby; therefore, some expensive things are shabby.” This inference (argument) may be symbolized as:

1. (∀x)[Hx ⊃ (Ex • Dx)] (“For all x, if x is a hotel, then x is expensive and x is depressing”)
2. (∃x)(Hx • Sx)
   Therefore,
3. (∃x)(Ex • Sx)

The four quantification rules (EG, EI, UG, and UI) that apply to syllogisms are applicable here as well. The finite-universe method of proving invalidity that applies to syllogisms is applicable here as well. See syllogism.
asymmetry. A relation such that if one thing has that relation to a second, then the second cannot have that relation to the first. Symbolically: \((x)(y)(Rxy \rightarrow \neg Ryx)\). Examples: “is the father of,” “is north of,” “is older than,” “weighs more than,” “is a child of.” See (1) nonsymmetry, (2) relation, and (3) symmetry.

attribute (a.k.a. predicate). In predicate logic, a property, feature, quality, or characteristic of an individual. Examples: “is human,” “is mortal,” “is beautiful.” Attributes are denoted by upper-case letters “A” through “Z.” Attribute variables are denoted by upper-case Greek letters “Ø” and “Ψ.” See (1) Greek letters “Ø” (phi) and “Ψ” (psi) and (2) individual.

axiom of replacement. See replacement, axiom of.

Barbara. In categorical logic, a standard-form categorical syllogism with mood and figure AAA-1. All three of its propositions are “A” propositions, and it is in the first figure because the middle term is the subject of the major premise and the predicate of the minor premise. The syllogism may be reconstructed as follows:

1. All M are P.
2. All S are M.
   Therefore,
3. All S are P.

The syllogism is unconditionally valid—one of 15 standard-form categorical syllogisms with that characteristic. See (1) figure, (2) mnemonic terms, (3) mood, and (4) standard-form categorical syllogism (SFCS).

biconditional. In propositional logic, a truth-functional compound proposition formed by putting “if and only if” between two propositions. The symbol for a biconditional is the triple bar (tribar) (“≡”). A biconditional is true just in case either (1) its two component propositions are true or (2) its two component propositions are false. The relation expressed by a biconditional is material equivalence. See (1) conditional, (2) material equivalence, and (3) truth-functional compound proposition.

binary (dyadic) relation. A relation that holds (obtains) between two individuals, i.e., a two-place relation. For example, “Ron is married to Nancy,” “Dallas is north of Houston,” and “Cain was brother to Abel.” See relation.

bivalence, law (principle) of. The law of classical logic that every proposition is either true or false. That is, there are just two values a proposition may take: ‘true’
and ‘false’. Another way to put this is that the truth values ‘true’ and ‘false’ are jointly exhaustive; i.e., there is no third or middle possibility. The law of bivalence is not to be confused with the law of excluded middle, which asserts that every proposition is either true or not true.

<table>
<thead>
<tr>
<th>Law of excluded middle</th>
<th>Every proposition is either true or not true (this is an instance of the more general law that every object either has or lacks a given property)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Law of bivalence</td>
<td>Every proposition is either true or false</td>
</tr>
</tbody>
</table>

See excluded middle, law (principle) of.

**Boole, George** (born 2 November 1815, Lincoln, Lincolnshire, England—died 8 December 1864, Ballintemple, County Cork, Ireland). English mathematician who helped establish modern symbolic logic and whose algebra of logic, now called Boolean algebra, is basic to the design of digital computer circuits (from *Encyclopædia Britannica* online). See Boolean symbolism.

**Boolean equations.** See Boolean symbolism.

**Boolean interpretation of standard-form categorical propositions.** See (1) Boole, George and (2) square of opposition.

**Boolean symbolism.** A way of representing standard-form categorical syllogisms (“A,” “E,” “I,” and “O”) as equalities and inequalities, to wit:

- The “A” proposition, ‘All S are P’, is symbolized as \( S \cap P = 0 \).
- The “E” proposition, ‘No S are P’, is symbolized as \( S \cap \overline{P} = 0 \).
- The “I” proposition, ‘Some S are P’, is symbolized as \( S \cap P \neq 0 \).
- The “O” proposition, ‘Some S are not P’, is symbolized as \( S \cap \overline{P} \neq 0 \).

See Boole, George.

**bound variable.** In predicate logic, a variable that is bound by a quantifier. See (1) free variable and (2) quantifier.

**categorical logic (a.k.a. classical logic, syllogistic logic, and Aristotelian logic).** The logic of categories or classes. This type of logic concerns relations of class inclusion (either total or partial) and class exclusion (either total or partial). Various means (such as the Square of Opposition and Venn diagrams) have been
devised to determine whether particular categorical syllogisms are valid. See (i) Aristotle, (2) predicate logic, and (3) propositional logic.

categorical proposition. A proposition about classes (categories), affirming or denying that a class S is included in a class P, either in whole or in part. See (i) proposition and (2) standard-form categorical proposition (SFCP).

categorical syllogism. A syllogism consisting of three categorical propositions that together contain three terms, each of which occurs in exactly two of the constituent propositions. See (i) categorical proposition, (2) standard-form categorical syllogism (SFCS), and (3) syllogism.

category. See class.

chain argument. A series of two or more arguments, with the conclusion of the first argument serving as a premise of a second argument, the conclusion of the second argument serving as a premise of a third argument, and so on, for as many arguments as there are. Like a physical chain, a chain argument is no stronger than its weakest link. See argument.

change-of-quantifier rule (CQ). In predicate logic, a set of four logical equivalences that allow for the replacement of (i) a universal quantifier with an existential quantifier or (2) an existential quantifier with a universal quantifier. The rule is that one quantifier may be replaced with the other, provided that tildes are placed before and after the quantifier. Here are the four logical equivalences:

- ‘(x)Øx‘ :: ‘~(∃x)Øx’
- ‘(∃x)Øx‘ :: ‘~(x)Øx’
- ‘(x)Øx‘ :: ‘~(∃x)Øx’
- ‘(∃x)Øx‘ :: ‘~(x)Øx’

Note that double negation is silently employed. See (i) double negation and (2) quantifier.

Chrysippus (born c. 280 BCE—died c. 206). Greek philosopher from Soli (Soloi) who was the principal systematizer of Stoic philosophy. He is considered to have been, with Zeno, cofounder of the academy at Athens Stoa (Greek: “Porch”). Credited with about 750 writings, he was among the first to organize propositional logic as an intellectual discipline (from Encyclopedia Britannica online). See propositional logic.
**class.** A collection (group, aggregate) of individuals (objects) that have some specified characteristic (property) in common, the characteristic being known as the class-defining characteristic.

**class complement.** See complement.

**cogency.** A property of (some) inductive arguments. An inductive argument is cogent if and only if (1) it is strong and (2) it has true premises. Cogency is the inductive analogue of deductive soundness. Cogency, like strength, is a matter of degree; it is not, like validity or soundness, all or nothing. See (1) induction, (2) strength, and (3) uncogency.

**commutation (Com).** In propositional logic, two replacement rules. The first rule says that the disjunctions of a disjunction may be switched (i.e., rearranged). Formally, ‘(p ∨ q)’ :: ‘(q ∨ p)’. The second rule says that the conjuncts of a conjunction may be switched. Formally, ‘(p • q)’ :: ‘(q • p)’.

**compactness.** A property of (some) deductive systems. A deductive system is compact if and only if (1) it is complete and (2) if even one of its rules of inference is eliminated, it becomes incomplete. See (1) completeness, (2) consistency 2, and (3) deduction.

**complement (a.k.a. complementary class, negative, and contradictory).** In categorical logic, the collection of all things that do not belong to a given class. Example: the complement of the class of people is the class of all things that are not people. The complement of a term (as opposed to a class) is formed by prefixing “non” to it. Thus, “nonpeople,” however odd it may be as a linguistic item, is the complement of “people.” The complement of “nonpeople,” however, is “people,” not “nonnonpeople.” (Double negation is silently employed.) See class.

**completeness.** A property of (some) deductive systems. A deductive system is complete if and only if all valid arguments are provable in it. See (1) compactness, (2) consistency 2, and (3) deduction.

**component.** In propositional logic, x is a component of a proposition if and only if (1) x is a proposition in its own right and (2) if x is replaced in the larger proposition with any other proposition, the result of that replacement is meaningful. For example, in the proposition “The man who shot Lincoln was an actor,” the final four words (“Lincoln was an actor”) satisfy the first requirement, but not the second; therefore, “Lincoln was an actor” is not a component of the larger proposition. See (1) compound proposition and (2) simple proposition.
**compound proposition.** In propositional logic, a proposition that (1) contains another proposition as a component and (2) remains *meaningful* when the component is replaced with any other proposition. The components themselves may be compound propositions. See (1) component and (2) simple proposition.

**conclusion.** In an argument, the proposition that is affirmed on the basis of the other propositions (the premises) of the argument. That which is inferred from the premises of a given argument. See (1) argument, (2) premise, and (3) proposition.

**conclusion indicator.** A word or phrase that indicates (but does not guarantee) that what follows it is the conclusion of an argument. Examples: “therefore,” “hence,” “thus,” “so,” “it follows that,” “consequently.” See (1) argument, (2) conclusion, and (3) premise indicator.

**condition.** See (1) necessary condition, (2) necessary and sufficient condition, and (3) sufficient condition.

**conditional (a.k.a. hypothetical proposition and implicative proposition).** In propositional logic, a truth-functional compound proposition formed by putting “only if” between two propositions. Stated differently, a truth-functional compound proposition expressed by an “if-then” sentence. A conditional contains two component propositions, each of which has a name. The component following the word “if” is the antecedent; the component following the word “then” is the consequent. The symbol for a conditional is the horseshoe (“⊃”). A conditional is true unless both (1) its antecedent is true and (2) its consequent is false. Thus, ‘p ⊃ q’ is an abbreviation for (and is logically equivalent to) ‘¬(p • ¬q)’. In other words, a conditional is true if either (1) its antecedent is false or (2) its consequent is true. Thus, ‘p ⊃ q’ is an abbreviation for (and is logically equivalent to) ‘¬p ∨ q’. The relation expressed by a conditional is material implication. See (1) antecedent, (2) biconditional, (3) consequent, (4) material implication, and (5) truth-functional compound proposition.

**conditional proof.** A method of proof that consists of (1) assuming the antecedent of a required conditional on the first line of an indented sequence, (2) deriving the consequent of the required conditional on a subsequent line, using only valid rules of inference, and (3) discharging the indented sequence in a conditional that exactly replicates the one to be obtained. See (1) conditional and (2) proof.

**conditional validity.** A property of (some) standard-form categorical syllogisms (SFCSs). A conditionally valid argument is an argument that is valid on the condition
that (i.e., if and only if) at least one member of a given class exists. See (1) unconditional validity and (2) validity.

conjuncts. The propositions (simple or compound) that make up a conjunction. In the conjunction “Baseball is a sport and chess is not a sport,” the conjuncts are “Baseball is a sport” (a simple proposition) and “Chess is not a sport” (a compound proposition). Conjuncts can be referred to as “left” and “right” or as “first” and “second.” Thus, “Baseball is a sport” is the first or left conjunct of the aforementioned conjunction, while “Chess is not a sport” is the second or right conjunct. See (1) conjunction and (2) proposition.

conjunction.

1. (a.k.a. conjunctive proposition) A truth-functional propositional form. A truth-functional compound proposition formed by putting “and” between two propositions, which are called conjuncts. The symbol for conjunction is the dot (•). A conjunction is true just in case both of its conjuncts are true. See (1) conjuncts and (2) truth-functional compound proposition.

2. In propositional logic, a rule of inference (i.e., an elementary valid argument form, abbreviated as Conj) in which the conclusion is the conjunction of the argument’s two premises. Formally, ‘p’; ‘q’; therefore, ‘p • q’. See (1) elementary valid argument form and (2) inference, rules of.

connective, logical. See operator, logical.

connective, truth-functional. In propositional logic, a symbol that either precedes a proposition or joins two propositions so as to produce a truth-functional compound proposition. There are five truth-functional connectives: the tilde (~); the dot (•); the wedge (Ú); the horseshoe ( deberá); and the triple bar (tribar) (“≡”). See truth-functional compound proposition.

consequent (a.k.a. apodosis). In propositional logic, the part of a conditional that follows the word “then,” or, in the case of a symbolized expression, follows the horseshoe. Example: In the conditional “If this is an even-numbered year, then there are Congressional elections this year,” the consequent is “there are Congressional elections this year.” In the symbolized expression ‘q deberá p’, the consequent is ‘p’. See (1) antecedent and (2) conditional.

consistency.

1. (a.k.a. joint satisfiability) A logical relation between (among) propositional
forms (or propositions). Propositional form X is consistent with propositional form Y (i.e., X and Y are consistent [with one another]) if and only if it is logically possible for both X and Y to be true. Colloquially, X and Y can both be true. Example: ‘p • q’ is consistent with ‘p ∨ q’, since, in the case in which both ‘p’ and ‘q’ are true, both compound propositions are true. One may also speak of a set of propositional forms (or propositions) being consistent or inconsistent. See inconsistency.

2. A property of (some) deductive systems. A deductive system is consistent if and only if only valid arguments are provable in it. See (1) compactness and (2) completeness.

**constant.** In predicate logic, a symbol (lowercase “a” through “w”) that names (denotes, refers to, picks out) an individual. Examples: “a” denotes Alice; “b” denotes Boston; “c” denotes Colorado. See (1) individual and (2) variable.

**constructive dilemma (CD).** In propositional logic, a rule of inference (i.e., an elementary valid argument form) in which the first premise is a conjunction of conditionals, the second premise a disjunction of the antecedents of those conditionals, and the conclusion a disjunction of the consequents of those conditionals. Formally, ‘(p ⊃ q) • (r ⊃ s)’; ‘p ∨ r’; therefore, ‘q ∨ s’. See (1) destructive dilemma (DD), (2) elementary valid argument form, and (3) inference, rules of.

**contingency (a.k.a. syntheticity).** A logical property of propositional forms (or propositions). Propositional form X is contingent (i.e., X is a contingent propositional form) if and only if (1) it is logically possible for X to be true and (2) it is logically possible for X to be false. In other words, X is neither necessarily false nor necessarily true. Example: ‘p • q’ is contingent. See (1) contingent proposition and (2) contingent propositional form.

**contingent proposition.** Any proposition whose specific form is contingent. For example, “If it’s raining, then the game is canceled” is a contingent proposition, since its specific form, ‘p ⊃ q’, is contingent. See (1) contingency, (2) contingent propositional form, and (3) specific form 2.

**contingent propositional form.** A propositional form that has both true and false substitution instances. A propositional form that is neither necessarily true nor necessarily false. In other words, a propositional form that is both possibly false and possibly true. Example: ‘p • q’. See (1) contingency, (2) contingent proposition, (3) propositional form, (4) self-consistent propositional form, (5) self-contradictory propositional form, and (6) tautologous propositional form.
contingent truth. A proposition that is true, but not necessarily so. Examples: “Abraham Lincoln was the 16th president”; “the Boston Red Sox won the 2013 World Series”; “some bachelors are bald.” See necessary truth.

contradictoriness (a.k.a. denial). A logical relation between two propositional forms (or propositions). Propositional form X is the contradictory of propositional form Y (i.e., X and Y are contradictories of one another) if and only if (1) it is logically impossible for both X and Y to be true and (2) it is logically impossible for both X and Y to be false. In other words, X and Y necessarily have different truth values. Colloquially, X and Y can’t both be true; X and Y can’t both be false. Example: ‘p • q’ is the contradictory of ‘~p ∨ ~q’. See contradictories.

contradictories. Two propositional forms (or propositions) are contradictories of one another if and only if they stand in the logical relation of contradictoriness to one another. In categorical logic, “A” and “O” propositions are contradictories, as are “E” and “I” propositions. See contradictoriness.

contraposition. In categorical logic, an immediate inference that proceeds by (1) replacing the proposition’s subject term with the complement of its predicate term and (2) replacing the proposition’s predicate term with the complement of its subject term. The initial proposition is known as the premise; the resulting proposition is known as the contrapositive. Contraposition is logically equivalent to obversion, conversion, and obversion, in that order (use the acronym “OCO”). Thus, ‘All S are P’ becomes, successively, ‘No S are nonP’ (by obversion), ‘No nonP are S’ (by conversion), and ‘All nonP are nonS’ (by obversion). ‘Some S are not P’ becomes, successively, ‘Some S are nonP’ (by obversion), ‘Some nonP are S’ (by conversion), and ‘Some nonP are not nonS’ (by obversion).

<table>
<thead>
<tr>
<th>Premise</th>
<th>Contrapositive</th>
</tr>
</thead>
<tbody>
<tr>
<td>All S are P</td>
<td>All nonP are nonS</td>
</tr>
<tr>
<td>No S are P</td>
<td>No nonP are nonS</td>
</tr>
<tr>
<td>Some S are P</td>
<td>Some nonP are nonS</td>
</tr>
<tr>
<td>Some S are not P</td>
<td>Some nonP are not nonS</td>
</tr>
</tbody>
</table>

See immediate inference.

contraposition per accidens (a.k.a. contraposition by limitation). In categorical logic, an immediate inference in which one contraposes an “E” proposition. From ‘No S are P’, one infers ‘Some nonP are not nonS’. The inference is valid only on the traditional (Aristotelian) interpretation of categorical propositions. The inference is logically equivalent to subalternation and contraposition, in that order. Thus, ‘No S are P’ becomes ‘Some S are not P’ (by subalternation), which becomes
‘Some nonP are not nonS’ (by contraposition). See (1) contraposition, (2) conversion per accidens, and (3) immediate inference.

**contrapositive.** The conclusion of an immediate inference by contraposition. See (1) contraposition, (2) immediate inference, and (3) premise.

**contraries.** Two propositional forms (or propositions) are contraries (of one another) if and only if they stand in the logical relation of contrariety to one another. In categorical logic, “A” and “E” propositions are contraries on the traditional (Aristotelian) interpretation but not on the modern (Boolean) interpretation. See (1) contrariety and (2) subcontraries.

**contrariety.** A logical relation between two propositional forms (or propositions). Propositional form X is the contrary of propositional form Y (i.e., X and Y are contraries [of one another]) if and only if (1) it is logically impossible for both X and Y to be true and (2) it is logically possible for both X and Y to be false. Colloquially, X and Y can’t both be true; X and Y can both be false. Example: ‘p • q’ is the contrary of ‘¬p • ¬q’. See contraries.

**converse.**

1. In categorical logic, the conclusion of an immediate inference by conversion. See (1) conversion, (2) convertend, and (3) immediate inference.

2. In predicate logic, the converse of a relation R is the relation that holds between the objects b and a (in that order) whenever the relation R holds between a and b (in that order). Examples: “less than” is the converse (i.e., flip side) of “greater than”; “parent of” is the converse of “child of”; “is loved by” is the converse of “loves.” Note that “father of” is not the converse of “son of,” because the child in question may be a daughter rather than a son.

**conversion.** In categorical logic, an immediate inference that proceeds by interchanging the subject and predicate terms of the proposition. The initial proposition is known as the convertend; the resulting proposition is known as the converse.

<table>
<thead>
<tr>
<th>Convertend</th>
<th>Converse</th>
</tr>
</thead>
<tbody>
<tr>
<td>All S are P</td>
<td>All P are S</td>
</tr>
<tr>
<td>No S are P</td>
<td>No P are S</td>
</tr>
<tr>
<td>Some S are P</td>
<td>Some P are S</td>
</tr>
<tr>
<td>Some S are not P</td>
<td>Some P are not S</td>
</tr>
</tbody>
</table>

See immediate inference.
conversion per accidens (a.k.a. conversion by limitation). In categorical logic, an immediate inference in which one converts an “A” proposition. From ‘All S are P’, one infers ‘Some P are S’. The inference is valid only on the traditional (Aristotelian) interpretation of categorical propositions. The inference is logically equivalent to subalternation and conversion, in that order. Thus, ‘All S are P’ becomes ‘Some S are P’ (by subalternation), which becomes ‘Some P are S’ (by conversion). See (i) contraposition per accidens, (2) conversion, and (3) immediate inference.

convertend. The premise of an immediate inference by conversion. See (i) converse, (2) conversion, and (3) immediate inference.

copula. Some form of the verb “to be.” The copula connects (joins) the subject and predicate terms of a standard-form categorical proposition. (Note: In this course, for the sake of uniformity, we use “are” and “are not” as copulas, rather than, say, “is” and “is not.”) See standard-form categorical proposition (SFCP).

correlation. A principle by which relations are classified. Some relations, such as “A is a creditor of B,” are many-many, since many individuals can be a creditor of B and many individuals can have A as a creditor. Some relations, such as “A is the son of B,” are many-one, since many individuals can be the son of B, but only one individual can have A as a son. Some relations, such as “A is the father of B,” are one-many, since only one individual can be the father of B, but many individuals can have A as their father. Some relations, such as “A is greater by one than B,” are one-one, since only one individual can be greater by one than B, and only one individual can be such that A is greater by one than it is. Here is a summary:

<table>
<thead>
<tr>
<th>Many</th>
<th>One</th>
</tr>
</thead>
<tbody>
<tr>
<td>Many</td>
<td>“is a creditor of”</td>
</tr>
<tr>
<td>One</td>
<td>“is the father of”</td>
</tr>
</tbody>
</table>

corresponding conditional. To every argument form (or argument) there corresponds a conditional whose antecedent is the conjunction of the argument form’s premises and whose consequent is the argument form’s conclusion. For example, the corresponding conditional of the argument form ‘p ⊃ q; p’; therefore, ‘q’ (Modus Ponens) is ‘[(p ⊃ q) • p] ⊃ q’. An argument form is valid if and only if its corresponding conditional is a tautologous propositional form. See (i) argument form, (2) conditional, (3) Modus Ponens, and (4) tautologous propositional form.

counterdilemma. A way of rebutting (though not necessarily defeating or refuting) a constructive dilemma (CD). It consists of constructing a second constructive
dilemma whose conclusion is opposed (either as contrary or as contradictory) to the conclusion of the original. If the premises of the second (counter) dilemma are true, then the original dilemma has at least one false premise. See constructive dilemma (CD).

counterexample. An example that counters, or goes against, a proposition or an argument. For example, a black swan is a counterexample to the proposition that all swans are white. See refutation by logical analogy, method of.

counterexample method of refutation. See refutation by logical analogy, method of.

counterfactual. A counterfactual (usually indicated by the subjunctive mood) is a conditional of the form “if p were to happen q would,” or “if p were to have happened q would have happened,” where the supposition of p is contrary (counter) to the known fact that not-p. In truth-functional analysis, all counterfactuals are true, since their antecedents are, by definition, false. But some counterfactuals seem to be true, such as “If Joe Biden were a horse, then Joe Biden would have four legs,” and some false, such as “If Joe Biden were a horse, then Joe Biden would have five legs.” It follows that counterfactuals cannot (or should not) be interpreted as material conditionals of the sort dealt with in propositional logic. See conditional.

deduction. See deductive argument.

deductive argument (a.k.a. deduction). A type of argument in which the arguer claims that the conclusion follows necessarily (rather than probably) from the premise(s). The claim, in other words, is that, given the truth of the premises, it is logically impossible for the conclusion to be false. The claim, in other words, is that the premises logically imply the conclusion. Deductive arguments are either valid or invalid, depending on whether the claim made by the arguer is correct or incorrect. See (i) argument, (2) inductive argument, and (3) logical implication.

definiendum (Latin for “to be defined”). In a definition, the word or phrase to be defined. See (1) definiens and (2) definition.

definiens (Latin for “defining thing”). In a definition, the defining phrase or sentence. See (1) definiendum and (2) definition.

definition. A procedure for giving the meaning of a word or phrase. See (1) definiendum and (2) definiens.
definition symbol (“=df”). An abbreviation for “equals (i.e., means) by definition.” Example: “puppy” =df “young dog.” This may be read as “The word ‘puppy’ means, by definition, ‘young dog.’” See definition.

De Morgan, Augustus (born 27 June 1806, Madura, India—died 18 March 1871, London, England). English mathematician and logician whose major contributions to the study of logic include the formulation of De Morgan’s laws and work leading to the development of the theory of relations and the rise of modern symbolic, or mathematical, logic (from Encyclopedia Britannica online). See De Morgan’s theorems (DM).

De Morgan’s theorems (DM). In propositional logic, two replacement rules. The first rule says that the negation of the disjunction of two propositions is logically equivalent to the conjunction of the negations of the two propositions. Formally, ‘(~(p ∨ q)) ≣ ‘(~p • ~q). The second rule says that the negation of the conjunction of two propositions is logically equivalent to the disjunction of the negations of the two propositions. Formally, ‘(~(p • q)) ≣ ‘(~p ∨ ~q). See De Morgan, Augustus.

denying the antecedent (DA), fallacy of. In propositional logic, an invalid syllogism (i.e., a formal fallacy) in which the first premise is a conditional, the second premise the denial of the antecedent of that conditional, and the conclusion the denial of the consequent of that conditional. The name derives from the fact that the second premise denies the antecedent of the first premise. Formally, ‘(p ⊃ q); ‘~p; therefore, ‘~q. See (1) affirming the consequent (AC), fallacy of and (2) Modus Tollens.

destructive dilemma (DD). In propositional logic, a valid syllogism in which the first premise is a conjunction of conditionals, the second premise a disjunction of the denials of the consequents of those conditionals, and the conclusion a disjunction of the denials of the antecedents of those conditionals. Formally, ‘(p ⊃ q) • (r ⊃ s); ‘~q ∨ ~s; therefore, ‘~p ∨ ~r’. While valid, DD is not one of the eight rules of inference (i.e., elementary valid argument forms), for it can be reduced to two other rules: constructive dilemma (CD) and transposition (Trans). See (1) constructive dilemma (CD) and (2) transposition (Trans).

direct proof. See (1) indirect proof and (2) proof.

direct truth table. Another name for a full (as opposed to a partial) truth table. See (1) partial truth table and (2) truth table.
disjuncts (a.k.a. alternatives, alternants, and alternates). The propositions (simple or compound) that make up a disjunction. In “Joe is tired or Phil ate Wheaties,” the disjuncts are “Joe is tired” and “Phil ate Wheaties.” Disjuncts can be referred to as “left” and “right” or as “first” and “second.” Thus, “Joe is tired” is the first or left disjunct of the disjunction, while “Phil ate Wheaties” is the second or right disjunct. See disjunction.

disjunction (a.k.a. alternation and alternative proposition). A truth-functional propositional form. A truth-functional compound proposition formed by putting “or,” “unless,” or “if not” between two propositions, which are called disjuncts (or alternatives). The symbol for disjunction is the wedge (“∨”). A disjunction is true just in case at least one of its disjuncts is true. See (1) disjuncts and (2) truth-functional compound proposition.

disjunctive syllogism (DS) (a.k.a. modus tollendo ponens). In propositional logic, a rule of inference (i.e., an elementary valid argument form) in which the first premise is a disjunction, the second premise the negation of the first disjunct of the disjunction, and the conclusion the second disjunct of the disjunction. Formally, ‘p ∨ q’; ‘¬p’; therefore, ‘q’. See (1) elementary valid argument form and (2) inference, rules of.

distribution.

1. In categorical logic, a proposition distributes a term if and only if it (the proposition) refers to, or makes a claim about, all members of the class designated (denoted) by the term. Universal propositions distribute their subject term; negative propositions distribute their predicate term. Thus, an “A” proposition (universal affirmative) distributes only its subject term; an “E” proposition (universal negative) distributes both its subject term and its predicate term; an “I” proposition (particular affirmative) distributes neither its subject term nor its predicate term; an “O” proposition (particular negative) distributes only its predicate term.

<table>
<thead>
<tr>
<th>Subject Term Distributed</th>
<th>Predicate Term Distributed</th>
<th>Predicate Term Not Distributed</th>
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<tbody>
<tr>
<td>E</td>
<td></td>
<td>A</td>
</tr>
<tr>
<td>O</td>
<td></td>
<td>I</td>
</tr>
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</table>

2. In propositional logic, two replacement rules (abbreviated as Dist). The
first rule says that a proposition and a dot may be either distributed to or collected from each disjunct of a disjunction. Formally, ‘p • (q ∨ r) :: (p • q) ∨ (p • r). The second rule says that a proposition and a wedge may be either distributed to or collected from each conjunct of a conjunction. Formally, ‘p ∨ (q • r) :: (p ∨ q) • (p ∨ r). See replacement rules.

dot. In propositional logic, the symbol (‘•’) for conjunction. See conjunction 1.

double colon (‘::’). The symbol for logical equivalence. For example, ‘p :: ‘~~p’ means that the propositional form ‘p’ is logically equivalent to the propositional form ‘~~p’. The double colon has a different logical status from symbols such as ‘~,” “•,” “v,” “=” and “.” The latter are logical (i.e., truth-functional) operators on propositions (or propositional forms); the double colon, by contrast, is a metalogical symbol about propositions (or propositional forms). See logical equivalence.

double negation (DN). In propositional logic, a replacement rule. It says that tildes may be added or subtracted in pairs. Formally, ‘p :: ‘~~p’. See replacement rules.

elementary valid argument. Any argument that is a substitution instance of an elementary valid argument form (such as Modus Ponens). In other words, any argument that has a valid form is a valid argument. See elementary valid argument form.

elementary valid argument form. In propositional logic, any of eight argument forms (such as Modus Ponens) the conclusions of which follow logically from (i.e., are logically implied by) the premises. When propositions are substituted for the propositional variables—the same proposition being substituted for the same propositional variable throughout—the result is a valid argument. In general, an elementary valid argument form is an argument form the conclusion of which follows logically from the premise or premises. See (1) elementary valid argument and (2) rules of implication.

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end term. In categorical logic, a term that appears in the conclusion of a syllogism but not in either of its premises. Contrasted with “middle term.” The major term and the minor term are end terms.

<table>
<thead>
<tr>
<th></th>
<th>Terms</th>
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<tr>
<td></td>
<td>End</td>
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<tr>
<td>Major</td>
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<tr>
<td>Minor</td>
<td>3</td>
</tr>
</tbody>
</table>
See (1) major term, (2) middle term, and (3) minor term.

**entailment.** See logical implication.

**enthymeme.** An argument that is stated incompletely, part of it being “understood” and only “in the mind.” Example: All men are mortal; therefore, Socrates is mortal. (The part being understood is “Socrates is a man.”) In categorical logic, if the major premise is unstated (suppressed), the enthymeme is said to be of the first order. If the minor premise is unstated (suppressed), the enthymeme is said to be of the second order. If the conclusion is unstated (suppressed), the enthymeme is said to be of the third order. See argument.

**“E” proposition.** In categorical logic, a universal negative standard-form categorical proposition: ‘No S are P’. See (1) “A” proposition, (2) “I” proposition, (3) “O” proposition, and (4) standard-form categorical proposition (SFCP).

**equivalence.** Let “X” and “Y” be propositional forms. The following diagram displays (among other things) the difference between material equivalence and logical equivalence:

<table>
<thead>
<tr>
<th>Implication</th>
<th>Equivalence</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material</strong></td>
<td>As a matter of fact, the following state of affairs does not obtain: <strong>X is true and Y is false.</strong>&lt;br&gt; X materially implies Y.&lt;br&gt; ‘X ⊳ Y’ is true.</td>
</tr>
<tr>
<td><strong>Logical</strong></td>
<td>Logically, the following state of affairs cannot obtain: <strong>X is true and Y is false.</strong>&lt;br&gt; X logically implies Y.</td>
</tr>
</tbody>
</table>

1 Put differently, “As a matter of fact, X and Y have the same truth value.”
2 Put differently, “Logically, X and Y have the same truth value.”
‘$X \supset Y$’ is a tautology. ‘$X \equiv Y$’ is a tautology.

See (1) logical equivalence, (2) material equivalence, and (3) tautology.

**equivalence relation.** A relation that is transitive, symmetrical, and reflexive. Examples: “is congruent to”; “has the same number of members as”; “has the same weight as”; “is identical to.” See (1) reflexivity, (2) relation, (3) symmetry, and (4) transitivity.

**equivocation, fallacy of.** An informal fallacy that occurs when a term is used in different senses (equivocally, rather than univocally) in the same argument. Example:

1. Knowledge is power.
2. All power corrupts.
   Therefore,

The word “power” is being used in different senses. In premise 1, it means “power to (do something).” In premise 2, it means “power over (others).” In the first sense, premise 1 is true but premise 2 is false. In the second sense, premise 2 is true but premise 1 is false. Therefore, whichever sense is chosen, the argument is unsound. See (1) fallacy, (2) informal fallacy, and (3) unsoundness.

**escaping (going) between the horns of a dilemma.** A way of defeating or refuting a constructive dilemma (CD). It consists of rejecting the disjunctive premise. Obviously, this is impossible if the disjunctive premise is a tautology (‘$p \lor \neg p$’). See (1) constructive dilemma (CD), (2) grasping (taking) the dilemma (bull) by the horn(s), and (3) tautology.

**exceptive proposition.** A proposition (such as “All except employees are eligible”) that is a conjunction of categorical propositions rather than a single categorical proposition. The proposition mentioned is analyzed as (1) “All nonemployees are eligible” and (2) “No employees are eligible.” Together, these two propositions assert that the classes in question (employees and eligible persons) are complementary. Other phrases that are analyzed as exceptive propositions, besides ‘All except S are P’, are ‘Almost all S are P’, ‘Not quite all S are P’, and ‘All but a few S are P’. See conjunction.

**excluded middle, law (principle) of (a.k.a. tertium non datur).** One of the three laws (principles) of thought. It asserts that every object either has or lacks a
given property. Formally, \((\forall x)(\neg \neg x \vee \neg x)\). It follows from this law that every proposition either has or lacks the property of truth, i.e., that every proposition is either true or not true, i.e., that every proposition of the form \(p \vee \neg p\) is true. The law of excluded middle is not to be confused with the law of bivalence, which asserts that every proposition is either true or false.

<table>
<thead>
<tr>
<th>Law of excluded middle</th>
<th>Every proposition is either true or not true (this is an instance of the more general law that every object either has or lacks a given property).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Law of bivalence</td>
<td>Every proposition is either true or false.</td>
</tr>
</tbody>
</table>

See (1) bivalence, law (principle) of, (2) identity, law (principle) of, (3) laws (principles) of thought, and (4) noncontradiction, law (principle) of.

**exclusive (strong) disjunction.** In propositional logic, a disjunction that is true whenever exactly one of its disjuncts is true. If both disjuncts are true or both false, the disjunction is false. See (1) disjunction and (2) inclusive (weak) disjunction.

**exclusive “or.”** See exclusive (strong) disjunction.

**existential assumption, fallacy of.** The fallacy of assuming that a class has members if it is not asserted explicitly that it does. See fallacy.

**existential fallacy.** See existential assumption, fallacy of.

**existential generalization (EG).** In predicate logic, an operation (i.e., an elementary valid argument form) that consists of (1) introducing an existential quantifier immediately prior to a proposition, a propositional function, or another quantifier; and (2) replacing at least one occurrence of the constant or variable that appears in the proposition or propositional function with the variable that appears in the quantifier. Formally, ‘\(\exists a \neg a\)’; therefore, ‘\(\exists x \neg x\)’. In English, this says either (1) “Individual a-w is Ø; therefore, something is Ø” or (2) “Anything (i.e., any arbitrarily selected individual) is Ø; therefore, something is Ø.” See (1) elementary valid argument form and (2) existential quantifier.

**existential import.** A proposition is said to have existential import if it is typically uttered to assert the existence of objects of some specified kind. On the traditional (Aristotelian) interpretation of categorical propositions, all four standard-form categorical propositions (“A,” “E,” “I,” and “O”) have existential import. On the modern (Boolean) interpretation, only particular propositions (“I” and “O”)
have existential import. See (i) square of opposition and (z) standard-form categorical proposition (SFCP).

**existential instantiation (EI).** In predicate logic, an operation (i.e., an elementary valid argument form) that consists of removing an existential quantifier and replacing every variable bound by that quantifier with the same constant. The constant must be a new name that does not appear in any previous line of the proof (including the line that contains the final premise, a slash mark, and the conclusion). Formally, ‘(∃x)Øx’; therefore, ‘Øa-w’. In English, this says “Something is Ø; therefore, (let) individual a-w (be the individual that) is Ø.” See (i) elementary valid argument form and (z) existential quantifier.

**existential name.** The name given to the individual that is claimed to exist by an existentially quantified proposition. Existential instantiation (EI) consists of inferring ‘Øa’ from ‘(∃x)Øx’ (read as “Something is Ø, so let ‘a’ be its name”). The existential name in this case is “a.” See (i) existential instantiation (EI) and (z) instantial letter.

**existential quantifier.** In predicate logic, the symbol ‘(∃x)’, as in ‘(∃x)Øx’ (read as “There is at least one x such that x is Ø,” or, in better English, “Something is Ø”). The quantifier used to translate particular (“I” and “O”) propositions. See (i) quantifier and (z) universal quantifier.

**exportation (Exp).** In propositional logic, a replacement rule. It says that a conditional with a conjunction as its antecedent is logically equivalent to a conditional the antecedent of which is the first conjunct of the antecedent of the original proposition and the consequent of which is a conditional the antecedent of which is the second conjunct of the antecedent of the original proposition and the consequent of which is the consequent of the original proposition. Formally, ‘(p • q) ⊃ r’ :: ‘p ⊃ (q ⊃ r)’. See replacement rules.

**fallacy.** An argument (or argument form) that appears to be correct but is not. An argument (or argument form) that is psychologically attractive (plausible, alluring), and therefore commonly made, but which is logically infirm. An error or mistake in reasoning other than the employment of false premises. Fallacy is not the same as sophistry, which is the deliberate use of unsound reasoning for some ulterior purpose, such as deception, “winning” an argument, or undermining proper discussion. Every fallacy is either formal or informal, depending on whether it can be detected merely through an examination of the form or structure of the argument. If it can be so detected, then it is a formal fallacy; if it cannot be so detected, but requires attention to content or context, then it is an informal fallacy. See (i) formal
fallacy, (2) informal fallacy, and (3) paralogism.

**fallacy of affirming the consequent (AC).** See affirming the consequent (AC), fallacy of.

**fallacy of denying the antecedent (DA).** See denying the antecedent (DA), fallacy of.

**fallacy of equivocation.** See equivocation, fallacy of.

**fallacy of existential assumption.** See existential assumption, fallacy of.

**fallacy of illicit process of the major term (a.k.a. illicit major).** See illicit process of the major term, fallacy of.

**fallacy of illicit process of the minor term (a.k.a. illicit minor).** See illicit process of the minor term, fallacy of.

**fallacy of undistributed middle.** See undistributed middle, fallacy of.

**falsity (falsehood).** A property of (some) propositions. A proposition is false when it incorrectly describes how things are, and true otherwise. See (1) proposition and (2) truth.

**figure.** In categorical logic, the figure of a standard-form categorical syllogism indicates the position of the middle term in the premises—and therefore the “logical shape” of the syllogism. There are four figures:

1. The middle term is the subject term of the major premise and the predicate term of the minor premise.
2. The middle term is the predicate term of both premises.
3. The middle term is the subject term of both premises.
4. The middle term is the predicate term of the major premise and the subject term of the minor premise.

<table>
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<th>Figure</th>
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<tbody>
<tr>
<td>1</td>
<td>M</td>
<td>P</td>
<td>M</td>
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<td>S</td>
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<td>S</td>
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<tr>
<td>4</td>
<td>M</td>
<td>P</td>
<td>P</td>
<td>P</td>
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</table>

Each of the 64 moods (AAA through OOO) can occur with each of the four figures.
(1 through 4), which gives a total of 256 distinct standard-form categorical syllogisms. Example: AIO-3:

1. All M are P.
2. Some M are S.
   Therefore,
3. Some S are not P.

Only 24 of the 256 standard-form categorical syllogisms (9.3%) are valid: 15 of them unconditionally and nine conditionally (the condition being the existence of members of one of the three classes). See (1) mood and (2) standard-form categorical syllogism (SFCS).

**finite-universe method of proving invalidity.** In predicate logic, a method of proving the invalidity of an argument in which one shows that there is a possible universe or model containing at least one individual such that the argument’s premises are true and its conclusion false of that model. One begins with a one-member universe or model. If this model does not produce the desired result, then one moves to a two-member universe or model. If this model does not produce the desired result, then one moves to a three-member universe or model—and so on, until one either produces the desired result or gives up. Failing to prove invalidity is not equivalent to proving validity. The most one can say, having tried unsuccessfully to prove invalidity, is that the argument is probably valid, the degree of probability being a function of (1) how long and how hard one tried to prove invalidity and (2) one’s intelligence and ingenuity. See validity.

**formal fallacy.** A fallacy that can be detected merely by examination of the form or structure of the argument. See (1) fallacy and (2) informal fallacy.

**formal logic.** The type of logic in which the reasoning being studied is expressed in artificial (as opposed to natural) language. See (1) artificial symbolic language, (2) informal logic, (3) logic, and (4) natural language.

**formal proof (of validity).** A formal proof that a given argument (or argument form) is valid is a sequence of propositions each of which is either a premise of that argument or follows from preceding propositions of the sequence by an elementary valid argument, and the last proposition in the sequence is the conclusion of the argument whose validity is being proved. A formal proof of validity is effective in the sense that it can be mechanically decided of any given sequence whether it is a proof. Constructing a formal proof, by contrast, is not an effective procedure, for there is no guarantee that one will be able to “figure out” how to derive the conclusion from the premise(s), given the rules of inference. In this respect, formal
proof is unlike truth tables, which are completely mechanical. See (1) proof, (2) truth table, and (3) validity.

**free variable.** In predicate logic, a variable that is not bound by a quantifier. See (1) bound variable and (2) quantifier.

**Frege, Gottlob** (born 8 November 1848, Wismar, Mecklenburg-Schwerin—died 26 July 1925, Bad Kleinen, Germany). German mathematician and logician, who founded modern mathematical logic. Working on the borderline between philosophy and mathematics—viz., in the philosophy of mathematics and mathematical logic (in which no intellectual precedents existed)—Frege discovered, on his own, the fundamental ideas that have made possible the whole modern development of logic and thereby invented an entire discipline (from Encyclopedia Britannica online). See predicate logic.

**full truth table.** Another name for a truth table (contrasted with a partial truth table). See (1) partial truth table and (2) truth table.

**generalization (a.k.a. quantification).** In predicate logic, the formation of a proposition from a propositional function by placing a universal or existential quantifier before it. For example, the propositional function 'Jx' (“x is a jaywalker”) is generalized as the proposition ‘(x)Jx’ (“Everything is a jaywalker”) or the proposition '(∃x)Jx' (“Something is a jaywalker”). See (1) existential quantifier, (2) propositional function, (3) quantifier, and (4) universal quantifier.

**general proposition.** A quantified proposition, such as ‘All S are P’, ‘No S are P’, ‘Some S are P’, and ‘Some S are not P’. See singular proposition.

**grasping (taking) the dilemma (bull) by the horn(s).** A way of defeating or refuting a constructive dilemma (CD). It consists of rejecting the conjunctive premise, the two conjuncts of which are conditionals. To reject the first conditional (and therefore the conjunction as a whole) is to grasp the bull by the left horn; to reject the second conditional (and therefore the conjunction as a whole) is to grasp the bull by the right horn. One can, of course, grasp the bull by both horns. See (1) constructive dilemma (CD) and (2) escaping (going) between the horns of a dilemma.

**Greek letters “Ø” (phi) and “Ψ” (psi).** In predicate logic, symbols that represent predicates or attributes. Just as “x,” “y,” and “z” are variables for objects, “Ø” and “Ψ” are variables for attributes. The propositional function ‘Øx’ may be read as “x is Ø.” If “x” is replaced with “a,” the name for, say, Allen Buchanan, and “Ø”
is replaced with “P,” for the attribute “is a philosopher,” then we have the proposition ‘Pa,’ which says that Allen Buchanan is a philosopher (i.e., has the property of being a philosopher). Propositions, unlike propositional functions, are either true or false. (Propositional functions are neither true nor false; they lack truth value.) It is true that Allen Buchanan (one of my teachers at the University of Arizona during the 1980s) is a philosopher. See (i) attribute and (2) variable.

**horseshoe.** In propositional logic, the symbol ("\(\rightarrow\)") for material implication. See material implication 1.

**hypothetical.** See conditional.

**hypothetical syllogism (HS) (a.k.a. pure hypothetical syllogism).** In propositional logic, a rule of inference (i.e., an elementary valid argument form) in which (i) both premises and the conclusion are conditionals, (2) the antecedent of the first premise is identical to the antecedent of the conclusion, (3) the consequent of the first premise is identical to the antecedent of the second premise, and (4) the consequent of the second premise is identical to the consequent of the conclusion. Formally, ‘\(p \supset q\); ‘\(q \supset r\); therefore, ‘\(p \supset r\). See (i) elementary valid argument form and (2) inference, rules of.

**identity.** A relation that holds (obtains) only between an object and itself. To say that x and y are identical is to say that they are the same thing. Everything is identical with itself and with nothing else. Examples: Cicero is identical with (i.e., is the same thing as) Tully (symbolized as “Cicero = Tully”); the longest river in South America = the Amazon; Whitman = the author of *Leaves of Grass*; 13 = the sixth prime number. The relation of identity is transitive, symmetrical, and reflexive:

- Transitive: \((x)(y)(z)[[(x = y) \cdot (y = z)] \supset (x = z)]\)
- Symmetrical: \((x)(y)[(x = y) \supset (y = x)]\)
- Reflexive: \((x)(x = x)\)

See (1) reflexivity, (2) relation, (3) symmetry, and (4) transitivity.

**identity, law (principle) of.** One of the three laws (principles) of thought. It asserts that every object that has a given property has that property. Formally, \((x)(\Box x \supset \Box x)\). It follows from this law that every proposition that has the property of truth has the property of truth, i.e., that every proposition that is true is true, i.e., that every proposition of the form ‘\(p \supset p\)’ is true, i.e., that every proposition materially implies itself. See (1) excluded middle, law (principle) of, (2) laws (princi-
iff. An abbreviation for “if and only if.” See biconditional.

illicit contraposition. A formal fallacy that occurs when the conclusion of an argument depends on the contraposition of an “E” or an “I” proposition:

- No S are P; therefore, no nonP are nonS.
- Some S are P; therefore, some nonP are nonS.

Here are counterexamples (i.e., refutations by logical analogy):

- No gorillas are lions (true); therefore, no nonlions are nongorillas (false).
- Some animals are nondogs (true); therefore, some dogs are nonanimals (false).

See (1) contraposition, (2) fallacy, and (3) formal fallacy.

illicit contrariety. A fallacy that occurs when the conclusion of an argument depends on an incorrect application of the contrary relation.

- Suppose S's exist. Then “A” and “E” propositions are contraries, for, while it is possible for both propositions to be false, it is not possible for both propositions to be true. It follows that (1) if the “A” proposition is true, then the corresponding “E” proposition is false; and (2) if the “E” proposition is true, then the corresponding “A” proposition is false. This is a correct (i.e., nonfallacious) application of the contrary relation.
- Suppose S's do not exist. Then “A” and “E” propositions are not contraries, for it is possible for both propositions to be true (indeed, both are true). To infer the falsity of one from the truth of the other is to commit the fallacy of illicit contrariety.

See (1) contrariety and (2) fallacy.

illicit conversion. A formal fallacy that occurs when the conclusion of an argument depends on the conversion of an “A” or an “O” proposition:

- All S are P; therefore, all P are S.
- Some S are not P; therefore, some P are not S.
Here are counterexamples (i.e., refutations by logical analogy):

- All dogs are animals (true); therefore, all animals are dogs (false).
- Some vegetables are not carrots (true); therefore, some carrots are not vegetables (false).

See (1) conversion, (2) fallacy, and (3) formal fallacy.

**illicit major.** See illicit process of the major term, fallacy of.

**illicit minor.** See illicit process of the minor term, fallacy of.

**illicit process of the major term, fallacy of.** In a valid standard-form categorical syllogism, if an end term is distributed in the conclusion, then it must be distributed in the premises. Any syllogism in which the major term is distributed in the conclusion but not in the major premise commits the (formal) fallacy of illicit process of the major term (or, more briefly, illicit major). See (1) end term, (2) fallacy, (3) formal fallacy, (4) major term, and (5) standard-form categorical syllogism (SFCS).

**illicit process of the minor term, fallacy of.** In a valid standard-form categorical syllogism, if an end term is distributed in the conclusion, then it must be distributed in the premises. Any syllogism in which the minor term is distributed in the conclusion but not in the minor premise commits the (formal) fallacy of illicit process of the minor term (or, more briefly, illicit minor). See (1) end term, (2) fallacy, (3) formal fallacy, (4) minor term, and (5) standard-form categorical syllogism (SFCS).

**illicit subalternation.** A fallacy that occurs when the conclusion of an argument depends on an incorrect application of the subalternation relation.

- Suppose S's exist. Then (1) the “A” proposition is the superaltern of the corresponding “I” proposition, for (a) it is not possible for the “A” proposition to be true while the corresponding “I” proposition is false and (b) it is possible for the “I” proposition to be true while the corresponding “A” proposition is false; and (2) the “E” proposition is the superaltern of the corresponding “O” proposition, for (a) it is not possible for the “E” proposition to be true while the corresponding “O” proposition is false and (b) it is possible for the “O” proposition to be true while the corresponding “E” proposition is false. It follows that (1) if the “I” proposition is false, then
the corresponding “A” proposition is false and (2) if the “O” proposition is false, then the corresponding “E” proposition is false. This is a correct (i.e., nonfallacious) application of the subalternation relation.

- Suppose S’s do not exist. Then (1) the “A” proposition is not the superaltern of the corresponding “I” proposition, for it is possible for the “A” proposition to be true while the corresponding “I” proposition is false (indeed, the “A” proposition is true and the corresponding “I” proposition is false); and (2) the “E” proposition is not the superaltern of the corresponding “O” proposition, for it is possible for the “E” proposition to be true while the corresponding “O” proposition is false (indeed, the “E” proposition is true and the corresponding “O” proposition is false). To infer the truth of the “I” proposition from the truth of the corresponding “A” proposition, or the truth of the “O” proposition from the truth of the corresponding “E” proposition, is to commit the fallacy of illicit subalternation.

See (1) fallacy and (2) subalternation.

**illicit subcontrariety.** A fallacy that occurs when the conclusion of an argument depends on an incorrect application of the subcontrary relation.

- Suppose S’s exist. Then “I” and “O” propositions are subcontraries, for, while it is possible for both propositions to be true, it is not possible for both propositions to be false. It follows that (1) if the “I” proposition is false, then the corresponding “O” proposition is true; and (2) if the “O” proposition is false, then the corresponding “I” proposition is true. This is a correct (i.e., nonfallacious) application of the subcontrary relation.

- Suppose S’s do not exist. Then “I” and “O” propositions are not subcontraries, for it is possible for both propositions to be false (indeed, both are false). To infer the truth of one from the falsity of the other is to commit the fallacy of illicit subcontrariety.

See (1) fallacy and (2) subcontrariety.

**immediate inference.** An inference in which a conclusion is drawn from a single premise, without the mediation of a second premise. Examples include conversion, obversion, contraposition, simplification (Simp), and tautology (Taut). See (1) inference and (2) mediate inference.

**implication.** Let “X” and “Y” be propositional forms. The following diagram displays (among other things) the difference between material implication and logical implication:
<table>
<thead>
<tr>
<th>Implication</th>
<th>Equivalence</th>
</tr>
</thead>
</table>
| **Material** | As a matter of fact, neither of the following states of affairs obtains: **X is true and Y is false**; **Y is true and X is false**.³  
X materially implies Y.  
‘X ⊃ Y’ is true. | As a matter of fact, the following state of affairs does not obtain: **X is true and Y is false**.  
X materially implies Y.  
‘X ⊃ Y’ is true. | As a matter of fact, neither of the following states of affairs obtains: **X is true and Y is false**; **Y is true and X is false**.³  
X is materially equivalent to Y.  
‘X ≡ Y’ is true. |
| **Logical** | Logically, the following state of affairs cannot obtain: **X is true and Y is false**.  
X logically implies Y.  
‘X ⊃ Y’ is a tautology. | Logically, neither of the following states of affairs can obtain: **X is true and Y is false**; **Y is true and X is false**.⁴  
X logically implies Y.  
‘X ⊃ Y’ is a tautology. | X is logically equivalent to Y.  
‘X ≡ Y’ is a tautology. |

See (1) logical implication, (2) material implication, and (3) tautology.

**Implication rules.** In propositional logic, eight rules (*Modus Ponens, Modus Tollens, Hypothetical Syllogism, Disjunctive Syllogism, Constructive Dilemma, Simplification, Conjunction, and Addition*) that allow a particular propositional form to be inferred from one or more other specified propositional forms. In each case, the conjunction of the premises logically implies the conclusion (but not conversely). The rules (i.e., elementary valid argument forms) are applicable only to whole lines of a proof. See (1) elementary valid argument form, (2) inference, rules of, (3) logical implication, and (4) replacement rules.

**inclusive (weak) disjunction.** In propositional logic, a disjunction that is true whenever at least one of its disjuncts is true. The only time an inclusive (weak) disjunction is false is when both disjuncts are false. See (1) disjunction and (2) exclusive (strong) disjunction.

**inclusive “or.”** See inclusive (weak) disjunction.

³ Put differently, “As a matter of fact, X and Y have the same truth value.”  
⁴ Put differently, “Logically, X and Y have the same truth value.”
inconsistency (a.k.a. joint unsatisfiability). A logical relation between (among) propositional forms (or propositions). Propositional form X is inconsistent with propositional form Y (i.e., X and Y are inconsistent [with one another]) if and only if it is logically impossible for both X and Y to be true, i.e., if and only if at least one of the propositional forms (X or Y) is false. Colloquially, X and Y can’t both be true. Example: ‘p ⊨ q’ is inconsistent with ‘p • ~q’. One may also speak of a set of propositional forms (or propositions) being consistent or inconsistent. See consistency.

independence (a.k.a. irrelevance). A logical relation between two propositional forms (or propositions). Propositional form X is independent of propositional form Y (i.e., X and Y are independent [of one another]) if and only if (1) it is logically possible for X to be true while Y is false; (2) it is logically possible for Y to be true while X is false; (3) it is logically possible for both X and Y to be true; and (4) it is logically possible for both X and Y to be false. Colloquially, X can be true while Y is false; Y can be true while X is false; X and Y can both be true; X and Y can both be false. Example: ‘p’ and ‘p ⊢ q’ are independent of one another. See logical implication.

indicator. See (1) conclusion indicator and (2) premise indicator.

indirect proof (a.k.a. reductio ad absurdum). A method of proof that consists of (1) assuming the negation of a required proposition on the first line of an indented sequence, (2) deriving a self-contradiction on a subsequent line, and then (3) discharging the indented sequence by asserting the negation of the assumed proposition (i.e., by affirming the required proposition). Example: to prove ‘p’, assume ‘~p’ and derive ‘r • ~r’; then conclude ‘~~p’, which is logically equivalent to ‘p’. See (1) negation, (2) proof, and (3) self-contradiction.

indirect truth table. Another name for a partial truth table. The contrast is with a direct (full) truth table. See (1) partial truth table and (2) truth table.

individual. In predicate logic, any thing—such as a country, a city, or a person—of which an attribute can be meaningfully predicated. Examples: Argentina, Chicago, Barack Obama. Individuals are denoted by constants (as opposed to variables). See (1) attribute and (2) constant.

individual constant. See constant.

individual variable. See variable.
induction. See inductive argument.

inductive argument (a.k.a. induction). A type of argument in which the arguer claims that the conclusion follows probably (rather than necessarily) from the premise(s). The claim, in other words, is that, given the truth of the premises, it is improbable for the conclusion to be false. Inductive arguments are either strong or weak, depending on whether the claim made by the arguer is correct or incorrect. Strictly speaking, all inductive arguments are invalid, since, in an inductive argument, the truth of the premises is consistent with the falsity of the conclusion. See (1) argument, (2) consistency 1, (3) deductive argument, and (4) validity.

inference. A psychological (mental), and hence temporal, process by which one proposition is arrived at and affirmed on the basis of one or more other propositions, which are accepted as the starting point of the process. An inference may be expressed as an argument if the objective is to persuade someone. (One typically does not argue with oneself.) Do not confuse inference with logical implication. We make inferences; we do not make, but discover, logical implications. See (1) argument and (2) proposition.

inference, rules of. In propositional logic, a set of rules consisting of eight implication rules (i.e., elementary valid argument forms) and 10 replacement rules, for a total of 18 rules. The rules provide the means by which the conclusion of an argument may be derived from its premises. See (1) implication rules and (2) replacement rules.

informal fallacy. A fallacy that cannot be detected merely by examination of the form or structure of the argument. The content or context must also be examined. See (1) fallacy and (2) formal fallacy.

informal logic. The type of logic in which the reasoning being studied is expressed in natural (as opposed to artificial) language. See (1) artificial symbolic language, (2) formal logic, (3) logic, and (4) natural language.

instantial letter. The letter (a variable or a constant) that is introduced by universal instantiation (UI) or by existential instantiation (EI). See (1) constant, (2) existential instantiation (EI), (3) existential name, (4) universal instantiation (UI), and (5) variable.

instantiation. In predicate logic, the formation of a proposition from a propositional function by substituting an individual constant for its individual variable. For example, the propositional function ‘Jx’ (“x is a jaywalker”) is instantiated as the
propositions ‘Ja’ (“Allen is a jaywalker”), ‘Jb’ (“Billy Bob is a jaywalker”), and ‘Je’ (“Carl is a jaywalker”). See (1) constant, (2) propositional function, and (3) variable.

**intransitivity.** A relation in which, if one thing has that relation to a second and the second has that relation to a third, then the first cannot have that relation to the third. Symbolically: $(x)(y)(z)[R_{xy} \land R_{yz} \supset \sim R_{xz}]$. Examples: “is the mother of,” “is the father of,” “is a child of,” “weighs exactly two pounds more than.” See (1) nontransitivity, (2) relation, and (3) transitivity.

**invalidity.** See validity.

**“I” proposition.** In categorical logic, a particular affirmative standard-form categorical proposition: ‘Some S are P’. See (1) “A” proposition, (2) “E” proposition, (3) “O” proposition, and (4) standard-form categorical proposition (SFCP).

**irreflexivity.** A relation that a thing cannot have to itself. Symbolically: $(x)\sim R_{xx}$. Examples: “is the parent of,” “is greater than,” “is north of,” “is married to,” “is not a member of the same family as.” See (1) nonreflexivity, (2) reflexivity, and (3) relation.

**irrelevance.** See independence.

**justification.** In a proof, the justification of a given line (proposition) consists of one or more numerals, which refer to the preceding propositions from which the given line is inferred, together with the abbreviation for the rule(s) of inference by which the given line follows from the preceding propositions. Strictly speaking, the justification is not part of the proof, but a supplement to it; it is a help and should always be included. See (1) inference, rules of and (2) proof.

**large scope.** See scope.

**law (principle) of excluded middle.** See excluded middle, law (principle) of.

**law (principle) of identity.** See identity, law (principle) of.

**law (principle) of noncontradiction (a.k.a. law (principle) of contradiction).** See noncontradiction, law (principle) of.

**laws of distribution.** In predicate logic, two replacement rules:

- ‘$(x)(\Theta x \land \Psi x)$’ :: ‘$(x)\Theta x \land (x)\Psi x$’. (Translation: “Everything is both $\Theta$ and $\Psi$”
is logically equivalent to “Everything is $\mathcal{O}$ and everything is $\Psi$.”

- ‘$\exists x (\mathcal{O} x \lor \psi x)$’ :: ‘$\exists x \mathcal{O} x \lor (\exists x) \psi x$’. (Translation: “Something is either $\mathcal{O}$ or $\psi$” is logically equivalent to “Either something is $\mathcal{O}$ or something is $\psi$.”)

The first rule says (in effect) that a universal quantifier may be either distributed to or collected from each of two conjuncts. The second rule says that an existential quantifier may be either distributed to or collected from each of two disjuncts. See replacement rules.

**laws (principles) of thought.** Three fundamental laws (principles) that are believed to be necessary and sufficient for “correct” or rational thinking. These laws (principles) are the Law of Identity, the Law of Noncontradiction, and the Law of Excluded Middle. See (1) excluded middle, law (principle) of, (2) identity, law (principle) of, and (3) noncontradiction, law (principle) of.

**lens.** In categorical logic, the part of a Venn diagram that is shaded by an “E” proposition. The term comes from geometry. See (1) “E” proposition, (2) lune, and (3) Venn diagram.

**logic.** From the Greek word “logos,” which could mean any of the following, depending on the context: statement, principle, law, reason, proportion, speech, thought, word, meaning, or explanation. “Logic” is defined variously as (1) the science (study) of logical implication, or valid inference; (2) the science (study) of the methods and principles used to distinguish good (correct) from bad (incorrect) reasoning; (3) the science (study) that evaluates arguments or argument forms; and (4) the normative study of reasoning. Based on this final definition, logic is concerned with categories 3 and 4 in the following diagram:

<table>
<thead>
<tr>
<th>Types of Study of Reasoning</th>
<th>Types of Reasoning</th>
<th>Practical (Issues in Action)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empirical (Descriptive/Explanatory) (The Real) (What 1s)</td>
<td>Theoretical (Issues in Belief)</td>
<td>1</td>
</tr>
<tr>
<td>Normative (Prescriptive/Justificatory) (The Ideal) (What Ought to Be)</td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

**logical connective.** See operator, logical.

**logical consequence.** See logical implication.
logical equivalence (a.k.a. equivalence, mutual logical implication, mutual strict implication, mutual entailment, and mutual logical consequence). A logical relation between two propositional forms (or propositions). Propositional form X is logically equivalent to propositional form Y (i.e., X and Y are logically equivalent to one another) if and only if (1) it is logically impossible for X to be true while Y is false and (2) it is logically impossible for Y to be true while X is false. In other words, (1) X logically implies Y and (2) Y logically implies X. In other words, X and Y necessarily have the same truth value. Colloquially, X can't be true while Y is false; Y can't be true while X is false. Example: ‘p ⊃ q’ is logically equivalent to ‘∼p ∨ q’. See (1) logical implication, (2) replacement rules, and (3) material equivalence.

logical implication (a.k.a. implication, strict implication, entailment, formal implication, tautologous implication, logical consequence, and deducibility). A logical relation between two propositional forms (or propositions). Propositional form X logically implies propositional form Y if and only if it is logically impossible for X to be true while Y is false. Colloquially, X can't be true while Y is false. Example: ‘p’ logically implies ‘p ∨ q’. See (1) implication rules, (2) material implication, and (3) validity.

logically undetermined truth value. A condition that exists or obtains when the truth value of a given propositional form cannot be determined solely by the truth value of some other propositional form. For example, if all one knows is that ‘p’ is true, then the truth value of ‘p • q’ is logically undetermined. See (1) truth value and (2) propositional form.

logical operator. See operator, logical.

lune. In categorical logic, the part of a Venn diagram that is shaded by an “A” proposition. The term comes from geometry. See (1) “A” proposition, (2) lens, and (3) Venn diagram.

main operator. In propositional logic, the operator (connective) in a compound proposition that has as its scope everything else in the proposition. For example, in ‘[(p ∨ ∼q) • (r • s)] • ∼(m ⊃ n)’, the main operator is the third dot from the left. The proposition is a conjunction, the left conjunct of which is itself a conjunction and the right conjunct of which is a negated conditional. The left conjunct is a conjunction of a disjunction and a conjunction. The right disjunct of the disjunction is a negation. See (1) compound proposition and (2) operator, logical.
**major premise.** In categorical logic, the premise that contains the major term. See (1) major term and (2) minor premise.

**major term.** In categorical logic, the term that occurs as the predicate of the conclusion. See (1) end term, (2) major premise, (3) middle term, and (4) minor term.

**material equivalence.**

1. A truth-functional propositional form. The relation expressed by a truth-functional biconditional. A truth-functional compound proposition formed by putting “if and only if” or “just in case” between two propositions. The symbol for material equivalence is the triple bar (tribar) (“≡”). Propositional form X is materially equivalent to propositional form Y if and only if (a) both X and Y are true or (b) both X and Y are false. Alternatively, propositional form X is materially equivalent to propositional form Y if and only if X and Y materially imply each other. See (1) biconditional, (2) logical equivalence, (3) material implication, and (4) truth-functional compound proposition.

2. In propositional logic, two replacement rules (abbreviated as ME). The first rule says that a biconditional may be replaced with a conjunction of the two conditionals (or vice versa). Formally, ‘p ≡ q’ :: (p ⊃ q) • (q ⊃ p). The second rule says that a biconditional may be replaced with a disjunction, the first disjunct of which is a conjunction of the two propositions making up the biconditional and the second disjunct of which is a conjunction of the negations of the two propositions making up the biconditional (or vice versa). Formally, ‘p ≡ q’ :: (p • q) ⊃ (¬p • ¬q). See replacement rules.

**material implication.**

1. A truth-functional propositional form. The relation expressed by a truth-functional conditional. A truth-functional compound proposition formed by putting “only if” between two propositions or by using the expression “if-then.” The symbol for material implication is the horseshoe (“⊃”). Propositional form X materially implies propositional form Y if and only if it is not the case that X is true while Y is false, i.e., if and only if either X is false or Y is true. See (1) conditional, (2) logical implication, (3) material equivalence, and (4) truth-functional compound proposition.

2. In propositional logic, a replacement rule (abbreviated as MI). It says two things: (1) that a conditional may be turned into a disjunction, provided that its antecedent is negated; and (2) that a disjunction may be turned into a conditional, provided that its first disjunct is negated. Formally, ‘p ⊃ q’ ::
material implication, paradoxes of. The horseshoe symbol (”⊃”) is defined as a material conditional, so ‘p ⊃ q’ means nothing more (or less) than ‘~(p • ~q)’. All this says is that, as a matter of fact, it is not the case that the antecedent is true while the consequent is false. There are two paradoxical implications of this definition:

- If a given proposition is true, then it is materially implied by any proposition whatever. Formally, ‘p ⊃ (q ⊃ p)’.
- If a given proposition is false, then it materially implies any proposition whatever. Formally, ‘~p ⊃ (p ⊃ q)’.

(Both of these are tautologous propositional forms.) The following proposition, therefore, is true: “If the moon is made of green cheese (F), then the earth is round (T).” The paradox is resolved when one realizes that material implication states no meaning relation between the antecedent and the consequent. Hence, the fact that the antecedent and the consequent have unrelated subject matter has no bearing on the truth of the conditional of which they are parts. See (1) material implication and (2) tautologous propositional form.

mediate inference. An inference in which a conclusion is drawn from two or more premises. In the case of a syllogism, the conclusion is supposed to be drawn from the first premise through the mediation of the second. Examples include categorical syllogisms (from categorical logic) and Modus Ponens (from propositional logic). See (1) immediate inference, (2) inference, and (3) syllogism.

method of refutation by logical analogy. See refutation by logical analogy, method of.

middle term. In categorical logic, the term that appears in both premises but not in the conclusion. Contrasted with “end term.” See (1) end term, (2) major term, and (3) minor term.

minor premise. In categorical logic, the premise that contains the minor term. See (1) major premise and (2) minor term.

minor term. In categorical logic, the term that occurs as the subject of the conclusion. See (1) end term, (2) major term, (3) middle term, and (4) minor premise.

mnemonic terms. Names—such as “Barbara,” “Celarent,” and “Ferison”—that
medieval logicians introduced for the valid syllogisms. See Barbara.

**modern (Boolean) square of opposition.** See square of opposition.

**modus ponens (MP)** (a.k.a. *modus ponendo ponens, affirming the antecedent, and mixed hypothetical syllogism*). In propositional logic, a rule of inference (i.e., an elementary valid argument form) in which the first premise is a conditional, the second premise the antecedent of that conditional, and the conclusion the consequent of that conditional. Formally, ‘\( p \supset q \); ‘\( p \); therefore, ‘\( q \). *Modus Ponens* can be thought of as asserting the following: “Anything materially implied by a truth is true.” See (1) elementary valid argument form and (2) inference, rules of.

**modus tollens (MT)** (a.k.a. *modus tollendo Tollens, denying the consequent, and mixed hypothetical syllogism*). In propositional logic, a rule of inference (i.e., an elementary valid argument form) in which the first premise is a conditional, the second premise the denial of the consequent of that conditional, and the conclusion the denial of the antecedent of that conditional. Formally, ‘\( p \supset q \); ‘\( \sim q \); therefore, ‘\( \sim p \). *Modus Tollens* can be thought of as asserting the following: “Anything that materially implies a falsehood is false.” See (1) elementary valid argument form and (2) inference, rules of.

**monadic predicate.** A predicate used to assign an attribute to individual things. See (1) attribute and (2) relational (polyadic) predicate.

**mood.** In categorical logic, the mood of a standard-form categorical syllogism is represented by three capital letters, the first of which names the form of the syllogism’s major premise, the second of which names the form of the syllogism’s minor premise, and the third of which names the form of the syllogism’s conclusion. Example: AEO. There are 64 moods, ranging from AAA to OOO. See (1) figure, (2) major premise, (3) minor premise, and (4) standard-form categorical syllogism (SFCS).

**mutual entailment.** See logical equivalence.

**mutual logical consequence.** See logical equivalence.

**mutual logical implication.** See logical equivalence.

**mutual strict implication.** See logical equivalence.

**\( n \)-ary (\( n \)-adic, polyadic) relation.** A relation that holds (obtains) between or
among $n$ individuals, i.e., an $n$-place relation. See relation.

**natural deduction.** A proof procedure by which the conclusion of a deductive argument is derived from the argument’s premises through the use of rules of inference. See (1) deductive argument, (2) inference, rules of, and (3) proof.

**natural language.** A language, such as English, French, modern Greek, ancient Greek, Latin, or Chinese, that arose gradually, typically over a long period of time, with contributions made by many people, as opposed to being created by a particular person or persons at a particular time for a particular purpose. See artificial symbolic language.

**necessary and sufficient condition.** A condition that is both necessary (essential) and sufficient (adequate) for something (else) to be the case. Examples: Being a young dog is both necessary and sufficient for being a puppy; receiving a final score of 90 or higher in this course is both necessary and sufficient for receiving an A in this course. Two propositions that are materially equivalent to one another are necessary and sufficient for one another, since they materially imply each other. The symbol for material equivalence is the triple bar (tribar) (“≡”), which is read as “if and only if.” Hence, “$p ≡ q$” may be read as “$p$ if and only if $q$” or as “$p$ is necessary and sufficient for $q$.” See (1) material equivalence, (2) necessary condition, and (3) sufficient condition.

**necessary condition.** The condition represented by the consequent in a conditional. Example: In the conditional “If $4 > 3$, then $4 > 2$,” four’s being greater than two is a necessary condition of four’s being greater than three. In the conditional ‘$p \supset q$’, ‘$q$’ is a necessary condition of ‘$p$’. See (1) conditional and (2) sufficient condition.

**necessary truth.** A proposition that cannot (logically) be false. Examples: “All green books in the room are green objects”; “two plus two equals four”; “no bachelor is married.” See contingent truth.

**negation (a.k.a. contradictory and denial).** A truth-functional propositional form. A truth-functional compound proposition formed by putting “not,” “it is false that,” or “it is not the case that” in front of a proposition. The symbol for negation is the tilde (“~”). A negation is true just in case the negated proposition is false. See truth-functional compound proposition.

**negative proposition.** In categorical logic, a proposition that asserts that one class is excluded from another, either totally or partially. A proposition that denies
class membership. If the claim is one of totality, then the proposition is universal ('No S are P). If the claim is one of partiality, then the proposition is particular ('Some S are not P). See (i) affirmative proposition, (2) “E” proposition, and (3) “O” proposition.

**nego.** Latin for “I deny.” In categorical logic, the letter names “E” and “O” come from the two vowels of the word “nego.” The “E” proposition is universal negative; the “O” proposition is particular negative. See (1) affirmo, (2) “E” proposition, and (3) “O” proposition.

**“no.”** The universal negative quantifier, as in ‘No S are P’. See (1) “all” and (2) “some.”

**noncontradiction, law (principle) of (a.k.a. law [principle] of contradiction).** One of the three laws (principles) of thought. It asserts that no proposition both has and lacks a given property. Formally, \((\forall x)(\neg \neg x \cdot \neg x)\). It follows from this law that no proposition both has and lacks the property of truth, i.e., that no proposition is both true and not true, i.e., that every proposition of the form ‘\((\neg p \cdot \neg p)\)’ is true, or, equivalently, that every proposition of the form ‘\(p \cdot \neg p\)’ is false. See (1) excluded middle, law (principle) of, (2) identity, law (principle) of, and (3) laws (principles) of thought.

**nonreflexivity.** A relation that is neither reflexive nor irreflexive. Examples: “loves,” “hates,” “criticizes.” See (1) irreflexivity, (2) reflexivity, and (3) relation.

**nonsymmetry.** A relation that is neither symmetrical nor asymmetrical. Examples: “is a brother of,” “weighs no more than,” “loves.” See (1) asymmetry, (2) relation, and (3) symmetry.

**nontransitivity.** A relation that is neither transitive nor intransitive. Examples: “is a friend of,” “is a brother of,” “is not a brother of,” “loves,” “is discriminably different from,” “has a different weight than.” See (1) intransitivity, (2) relation, and (3) transitivity.

**normal-form formula (NFF).** In predicate logic, a formula in which negation signs (if there are any) apply only to simple predicates. Examples:

<table>
<thead>
<tr>
<th>NFF</th>
<th>Not a NFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\forall x)(\neg Fx \lor Gx))</td>
<td>((\exists x)(Fx \land \neg Gx))</td>
</tr>
<tr>
<td>((\forall x)(Fx \leftrightarrow \neg Gx))</td>
<td>((\exists x)(Fx \land \neg Gx))</td>
</tr>
</tbody>
</table>
\((\exists x)(Fx \land Gx)\)    \((\forall x)(\neg Fx \lor \neg Gx)\)

See simple predicate.

**obverse.** The conclusion of an immediate inference by obversion. See (1) immediate inference, (2) obversion, and (3) obvertend.

**obversion.** In categorical logic, an immediate inference that proceeds by (1) changing the quality of the proposition and (2) replacing the predicate term with its complement. The initial proposition is known as the obvertend; the resulting proposition is known as the obverse.

<table>
<thead>
<tr>
<th>Obvertend</th>
<th>Obverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>All S are P</td>
<td>No S are nonP</td>
</tr>
<tr>
<td>No S are P</td>
<td>All S are nonP</td>
</tr>
<tr>
<td>Some S are P</td>
<td>Some S are not nonP</td>
</tr>
<tr>
<td>Some S are not P</td>
<td>Some S are nonP</td>
</tr>
</tbody>
</table>

See (1) complement, (2) immediate inference, (3) predicate term, and (4) quality.

**obvertend.** The premise of an immediate inference by obversion. See (1) immediate inference, (2) obverse, and (3) obversion.

**Ockham’s Razor (a.k.a. the principle of parsimony).** The principle, originally expressed in Latin as *entia non sunt multiplicanda praeter necessitatem*, that entities are not to be multiplied beyond necessity. Modern version: “Keep it simple, stupid.” See Ockham, William of.

**Ockham, William of** (also called William Ockham, Ockham also spelled Occam, by name Venerabilis Inceptor [Latin: “Venerable Enterpriser”], or Doctor Invincibilis [“Invincible Doctor”]) (born c. 1285, Ockham, Surrey?, England—died 1347/49, Munich, Bavaria [now in Germany]). Franciscan philosopher, theologian, and political writer, a late scholastic thinker regarded as the founder of a form of nominalism—the school of thought that denies that universal concepts such as “father” have any reality apart from the individual things signified by the universal or general term (from *Encyclopedia Britannica* online). See Ockham’s Razor.

**operator, logical (a.k.a. logical connective).** In propositional logic, a special symbol used to translate ordinary-language propositions. The five truth-functional logical operators/connectives are the tilde, the dot, the wedge, the horseshoe, and
the triple bar (tribar). See connective, truth-functional.


opposition. In categorical logic, two standard-form categorical propositions having the same subject and predicate terms are opposed to one another when they differ in quality or quantity or both. The square of opposition depicts (displays) the various types of opposition between “A,” “E,” “I,” and “O” propositions. “A” and “E” propositions are opposed in quality, with “A” being affirmative and “E” negative. “A” and “I” propositions are opposed in quantity, with “A” being universal and “I” particular. “A” and “O” propositions are opposed in both quantity and quality, with “A” being universal affirmative and “O” being particular negative. See (1) square of opposition and (2) standard-form categorical proposition (SFCP).

ordering relation. A relation that is transitive, asymmetrical, and irreflexive. Examples: “is greater than”; “is earlier than”; “is more intelligent than”; “is heavier than”; “is north of.” See (1) asymmetry, (2) irreflexivity, (3) relation, and (4) transitivity.

overlapping quantifiers. Quantifiers that lie within the scope of one another. Examples:

- \((\forall x)(\exists y)D_{xy}\) (“Everything is different from everything”)
- \((\exists x)(\forall y)D_{xy}\) (“Something is different from something”)
- \((\forall x)(\exists y)D_{xy}\) (“Everything is different from something [or other]”)
- \((\exists x)(\forall y)D_{xy}\) (“Something is different from everything”)

See (1) quantifier and (2) scope.

paradox of strict implication. See technical validity.

paradoxes of material implication. See material implication, paradoxes of.

paralogism. Any fallacious reasoning. See (1) fallacy and (2) reasoning.

partial truth table (a.k.a. indirect truth table). In propositional logic, a method of proving—without having to construct a full truth table (which might be unwieldy)—either (1) the validity or invalidity of an argument or (2) the consistency or inconsistency of a set of two or more propositions.
• For validity/invalidity, the method consists of assuming that the argument is invalid by assigning the truth value true to each premise and the truth value false to the conclusion. If this necessarily leads to a contradiction, then the assumption (of invalidity) is false, which means that the argument is valid. If it does not necessarily lead to a contradiction, then the argument is invalid (for it will have been shown that it is possible for the premises to be true while the conclusion is false).

• For consistency/inconsistency, the method consists of assuming that the set of propositions is consistent by assigning the truth value true to each proposition in the set. If this necessarily leads to a contradiction, then the assumption (of consistency) is false, which means that the set of propositions is inconsistent. If it does not necessarily lead to a contradiction, then the set of propositions is consistent (for it will have been shown that it is possible for all the propositions to be true).

See (1) indirect proof and (2) truth table.

**particular affirmative proposition.** In categorical logic, the “I” proposition, ‘Some S are P’. It asserts that one class is partially included (contained) in another. See “I” proposition.

**particular negative proposition.** In categorical logic, the “O” proposition, ‘Some S are not P’. It asserts that one class is partially excluded from another. See “O” proposition.

**particular proposition.** In categorical logic, a proposition that asserts that one class is partially included (contained) in or partially excluded from another class. If the claim is one of inclusion, then the proposition is affirmative (‘Some S are P’). If the claim is one of exclusion, then the proposition is negative (‘Some S are not P’). See universal proposition.

**Peirce, Charles Sanders (born 10 September 1839, Cambridge, Massachusetts—died 19 April 1914, near Milford, Pennsylvania).** American scientist, logician, and philosopher who is noted for his work on the logic of relations and on pragmatism as a method of research (from *Encyclopedia Britannica* online). See Peirce’s Law.

**Peirce’s Law.** The tautologous propositional form ‘[(p ⊃ q) ⊃ p] ⊃ p’. See (1) Peirce, Charles Sanders and (2) tautologous propositional form.
**practical reasoning.** Reasoning that culminates in action (as opposed to belief). The reasoning is designed to help the reasoner decide what to do. See (i) reasoning and (2) theoretical reasoning.

**predicate.** See attribute.

**predicate calculus (a.k.a. predicative calculus and functional calculus).** The logical theory of inferences involving quantifiers. See (i) predicate logic and (2) quantification.

**predicate logic (a.k.a. quantification theory).** The logic of predicates, or quantification. A type of logic that combines the distinctive features of categorical logic and propositional logic. Predicate logic facilitates analysis of arguments about individuals (e.g., Socrates), about properties of individuals (e.g., being a Greek philosopher), and about relations between individuals (e.g., Socrates being the teacher of Plato). See (i) categorical logic, (2) Frege, Gottlob, and (3) propositional logic.

**predicate term.** In a standard-form categorical proposition, the term that comes immediately after the copula. See (i) copula, (2) standard-form categorical proposition (SFCP), and (3) subject term.

**premise.**

1. In an argument, a proposition (there may be more than one) that is affirmed (or assumed) as providing support or reasons for accepting the conclusion. A member of the set of propositions, assumed for the course of an argument, from which a conclusion is inferred. See (i) argument, (2) conclusion, and (3) proposition.
2. In categorical logic, the premise of an immediate inference by contraposition. See (i) contraposition, (2) contrapositive, and (3) immediate inference.

**premise indicator.** A word or phrase that indicates (but does not guarantee) that what follows it is the premise of an argument. Examples: “since,” “because,” “for,” “as,” “for the reason that,” “inasmuch as.” See (i) conclusion indicator and (2) premise.

**proof.** Demonstration of validity (in the case of arguments) or truth (in the case of propositions). With regard to validity, a proof that a given argument is valid is a sequence of propositions each of which is either a premise of that argument or follows from preceding propositions of the sequence by an elementary valid argument form, and the final proposition in the sequence is the conclusion of the argument whose validity is being proved. See (i) indirect proof and (2) validity.
**proposition.** That which is asserted or denied by a declarative sentence. An object of belief ("I believe that aliens exist"), nonbelief ("I do not believe that aliens exist"), or disbelief ("I believe that aliens do not exist"). The meaning of a declarative sentence. The bearer of truth value. Every proposition is either true or false, and no proposition is both true and false. We may not know, in a particular case, which truth value a proposition has; for example, it is unknown whether Abraham Lincoln thought about his son Tad just moments before he (Lincoln) was shot, though it is either true or false that he did so. Synonym: statement. See sentence.

**propositional calculus.** The logical calculus (i.e., calculating machine) whose expressions are letters representing propositions, and special symbols representing operations on those propositions, to produce propositions of greater complexity. The operations are negation (symbolized by "¬"), conjunction ("•"), disjunction ("∪"), material implication (" nowrap="\Rightarrow"\)"), and material equivalence (" nowrap="\equiv"\)"). See propositional logic.

**propositional form.** Any sequence of symbols containing propositional variables and logical operators but no propositions, such that when propositions are substituted for the propositional variables—the same proposition being substituted for the same propositional variable throughout—the result is a proposition. See (1) operator, logical and (2) proposition.

**propositional function.** In predicate logic, an expression that (1) contains an individual variable and (2) becomes a proposition when either (a) an individual constant is substituted for the individual variable (this is known as instantiation) or (b) a universal or existential quantifier is placed before the expression (this is known as generalization). The expression that remains when a quantifier is removed from a proposition. An expression which contains one or more variables and which expresses a proposition when values are given to the variables. See (1) generalization and (2) instantiation.

**propositional logic.** The logic of propositions. Unlike categorical logic, which is concerned with relations between categories or classes of objects, propositional logic is concerned with propositions, which are either true or false. Propositions are represented by letters, which are combined by means of logical operators to form more complex symbolic representations. See (1) categorical logic, (2) Chrysippus, (3) predicate logic, and (4) proposition.

**propositional variable.** In propositional logic, a letter for which, or in place of which, a proposition may be substituted. For example, in the expression 'p ⊃ q', 'p'
and ‘q’ are propositional variables. To avoid confusion, we use lowercase letters from the middle part of the alphabet: ‘p’, ‘q’, ‘r’, ‘s’, and so forth. See proposition.

**punctuation.** In propositional logic, parentheses (“( . . . )”), brackets (“[ . . . ]”), and braces (“{ . . . }”) are used to disambiguate (i.e., eliminate the ambiguity of) ambiguous expressions. For example, ‘p • q ∨ r’ is ambiguous; it might mean a conjunction whose right conjunct is a disjunction (i.e., ‘p • (q ∨ r)’), or it might mean a disjunction whose left disjunct is a conjunction (i.e., ‘(p • q) ∨ r’). See (1) conjunction and (2) disjunction.

**quality.** A property of categorical propositions. The quality of a categorical proposition is either affirmative or negative, depending on whether class inclusion (complete or partial) is affirmed or denied by the proposition. If class inclusion is affirmed, the proposition is affirmative. If class inclusion is denied, the proposition is negative. See (1) categorical proposition and (2) quantity.

**quantification.** See generalization.

**quantification rules.** In predicate logic, a set of four elementary valid argument forms: existential generalization (EG), existential instantiation (EI), universal generalization (UG), and universal instantiation (UI). These argument forms permit the construction of proofs of validity for arguments (such as “All men are mortal; Socrates is a man; therefore, Socrates is mortal”) whose validity turns on the inner structures of noncompound propositions occurring in them. See (1) elementary valid argument form, (2) existential generalization (EG), (3) existential instantiation (EI), (4) universal generalization (UG), and (5) universal instantiation (UI).

**quantifier.** In categorical logic, the words “all,” “no,” and “some.” In predicate logic, the symbols “(x)” (universal quantifier) and “(∃x)” (existential quantifier). See (1) “all,” (2) existential quantifier, (3) “no,” (4) “some,” and (5) universal quantifier.

**quantifier convention.** In predicate logic, the convention (stipulation, agreement) that in any quantified proposition, the quantifier applies to the smallest particle that the punctuation permits. Thus, the following is a disjunction (a compound proposition), the first disjunct of which is a universally quantified proposition and the second disjunct of which is an existentially quantified proposition: (x)Øx ∨ (∃x)~Øx. The following is a universally quantified proposition (a simple proposition): (x)(Øx ⊃ ~Ψx). The first is read as “Either everything is Ø or something is not Ø.” The second is read as “No Ø are Ψ.” See quantifier.
quantity. A property of categorical propositions. The quantity of a categorical proposition is either universal or particular, depending on whether the proposition refers to all members or only some members of the class designated by its subject term. If the proposition refers to all members, it is universal. If the proposition refers to only some members, it is particular. See quality.

quaternary (tetradic) relation. A relation that holds (obtains) between or among four individuals, i.e., a four-place relation. For example, “Dave, Tom, Bill, and Margaret played bridge together,” “America bought Alaska from Russia for seven million dollars,” and “The Texas Rangers traded Ian Kinsler to the Detroit Tigers for Prince Fielder.” See relation.

reasoning. A special kind of thinking in which problems are solved or in which inference takes place, that is, in which conclusions are drawn from premises. All reasoning is thinking, but not all thinking is reasoning. Some reasoning culminates in action; this is known as practical reasoning. Some reasoning culminates in belief; this is known as theoretical reasoning. See (1) inference, (2) practical reasoning, and (3) theoretical reasoning.

reductio ad absurdum (a.k.a. reductio ad impossibile). The method of proving a proposition true by showing that its denial, together with other propositions assumed to be true, leads to (i.e., logically implies) a contradiction. See indirect proof.

reflexivity. A relation that a thing must have to itself. Symbolically: (x)Rxx. Examples: “is the same age as,” “is equal to,” “is congruent to,” “is as intelligent as,” “has the same color hair as,” “is a contemporary of,” “is a member of the same family as,” “is identical with.” See (1) irreflexivity, (2) nonreflexivity, and (3) relation.

refutation. Proof of the invalidity of an argument (or, in the case of a proposition, of its falsehood). Do not confuse refutation with rebuttal. A rebuttal is an attempt to refute. A refutation is a successful rebuttal. See validity.

refutation by logical analogy, method of. Any fallacious deductive argument can, in principle, be proved invalid by finding a second argument that has exactly the same form as the first and is known to be invalid by the fact that its premises are known to be true while its conclusion is known to be false. Failure to think of a refuting analogy does not prove the argument to be valid; it may simply reflect the limitations of one’s thinking, attention, or creativity. See (1) counterexample and (2) fallacy.

refute. See refutation.
**relation.** A way in which two or more things (objects) stand to one another. (The contrast is with objects having properties.) Here are nine important properties (characteristics) of binary (dyadic) relations:

<table>
<thead>
<tr>
<th>Relational Property</th>
<th>Symbolic Expression (Let “R” Be the Relation in Question)</th>
<th>Example</th>
<th>How to Read It (Let the Domain Be Persons)</th>
<th>Pithy Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetry</td>
<td>((x)(y)(\text{Rxy} \supset \text{Ryx}))</td>
<td>(x) is a sibling of (y)</td>
<td>For any two persons (x) and (y), if (x) is a sibling of (y), then (y) is a sibling of (x).</td>
<td>Siblinghood is symmetrical.</td>
</tr>
<tr>
<td>Asymmetry</td>
<td>((x)(y)(\text{Rxy} \supset \sim \text{Ryx}))</td>
<td>(x) is the father of (y)</td>
<td>For any two persons (x) and (y), if (x) is the father of (y), then (y) is not the father of (x).</td>
<td>Fatherhood is asymmetrical.</td>
</tr>
<tr>
<td>Nonsymmetry</td>
<td>-----</td>
<td>(x) is a brother of (y)</td>
<td>-----</td>
<td>Brotherhood is nonsymmetrical. (Suppose (x) is a brother of (y); (y) may or may not be a brother of (x).)</td>
</tr>
<tr>
<td>Transitivity</td>
<td>((x)(y)(z)(\text{Rxy} \cdot \text{Ryz} \supset \text{Rxz}))</td>
<td>(x) is older than (y)</td>
<td>For any three persons (x), (y), and (z), if (x) is older than (y) and (y) is older than (z), then (x) is older than (z).</td>
<td>Age is transitive.</td>
</tr>
<tr>
<td>Intransitivity</td>
<td>((x)(y)(z)(\text{Rxy} \cdot \text{Ryz} \supset \text{Rxz}))</td>
<td>(x) is the mother of (y)</td>
<td>For any three persons (x), (y),</td>
<td>Motherhood</td>
</tr>
</tbody>
</table>

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[47]
<table>
<thead>
<tr>
<th>Property</th>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nontransitivity</td>
<td>( Ryz ) ![!] ( \Leftrightarrow ) (~Rxz)</td>
<td>and ( z ), if ( x ) is the mother of ( y ) and ( y ) is the mother of ( z ), then ( x ) is not the mother of ( z ). is intransitive.</td>
</tr>
<tr>
<td>Nontransitivity</td>
<td>( x ) is a friend of ( y )</td>
<td>Friendship is nontransitive. (Suppose ( x ) is a friend of ( y ) and ( y ) is a friend of ( z ); ( x ) may or may not be a friend of ( z )).</td>
</tr>
<tr>
<td>Reflexivity</td>
<td>( (x)Rxx )</td>
<td>everybody is the same age as ( y ) Sameness of age is reflexive.</td>
</tr>
<tr>
<td>Irreflexivity</td>
<td>( (x)\sim Rxx )</td>
<td>Nobody is the parent of ( y ) Parenthood is irreflexive.</td>
</tr>
<tr>
<td>Nonreflexivity</td>
<td>( x ) loves ( y )</td>
<td>Love is nonreflexive. (Some people love themselves and some do not.)</td>
</tr>
</tbody>
</table>

See the entry for each listed term (e.g., symmetry) for elaboration and additional examples. See binary (dyadic) relation.

**Relational (polyadic) predicate.** A predicate that expresses a connection between (or among) two or more individuals. See monadic predicate.

**Replacement, axiom of.** An axiom (first principle) to the effect that logically equivalent expressions may replace (be substituted for) one another wherever they occur in a proof sequence. See (1) logical equivalence, (2) proof, and (3) replacement rules.
replacement rules. In propositional logic, 10 rules (De Morgan’s theorems, Commutation, Association, Distribution, Double Negation, Transposition, Material Implication, Material Equivalence, Exportation, and Tautology) that allow replacement of one propositional form with another form that is logically equivalent to it. The replacement may occur on a whole line or on any part of a line. See (1) implication rules, (2) inference, rules of, (3) logical equivalence, and (4) replacement, axiom of.

rhetorical question. A sentence that has the form of an interrogative (typically with a question mark) but which functions as an informative. Example: “Do you really believe that the Dallas Cowboys will win the 2018 Super Bowl?” (Meaning: The Dallas Cowboys will not win the 2018 Super Bowl.) See sentence.

rules and fallacies for syllogisms. See syllogistic rules and fallacies.

rules of inference. See inference, rules of.

rules of replacement. See replacement rules.

schema of a standard-form categorical proposition. A schema or skeleton has four parts: quantifier, subject term, copula, and predicate term. See (1) copula, (2) predicate term, (3) quantifier, (4) standard-form categorical proposition (SFCP), and (5) subject term.

scope. That to which a quantifier applies (or ranges over). For example, in “(x)Øx • (3x)Ψx,” the scope of the universal quantifier is “Øx.” In “(x)(Øx ⊃ Ψx),” the scope of the universal quantifier is “(Øx ⊃ Ψx).” The universal quantifier has small scope in the first example and large scope in the second example. See (1) existential quantifier and (2) universal quantifier.

self-consistency (a.k.a. satisfiability and consistency). A logical property of propositional forms (or propositions). Propositional form X is self-consistent (i.e., X is a self-consistent propositional form) if and only if it is logically possible for X to be true. In other words, X is not necessarily false. Example: ‘∼p’ is self-consistent. See (1) self-consistent proposition and (2) self-consistent propositional form.

self-consistent proposition. Any proposition whose specific form is self-consistent. For example, “If it’s raining, then the game is canceled” is a self-consistent proposition, since its specific form, ‘p ⊃ q’, is a self-consistent propositional form.
See (1) self-consistency and (2) self-consistent propositional form.

**self-consistent propositional form.** A propositional form that has at least one true substitution instance. A propositional form that is not necessarily false. In other words, a propositional form that is possibly true. Example: ‘p ⊃ ¬p’. See (1) contingent propositional form, (2) propositional form, (3) self-consistency, (4) self-consistent proposition, (5) self-contradictory propositional form, and (6) tautologous propositional form.

**self-contradiction (a.k.a. self-contradictory proposition).** Any proposition whose specific form is self-contradictory. For example, “It’s raining and it’s not raining” is a self-contradiction, since its specific form, ‘p • ¬p’, is a self-contradictory propositional form. A proposition that both asserts and denies some other proposition. See (1) self-contradictoriness and (2) self-contradictory propositional form.

**self-contradictoriness (a.k.a. unsatisfiability, inconsistency, and logical falsehood).** A logical property of propositional forms (or propositions). Propositional form X is self-contradictory (i.e., X is a self-contradiction) if and only if it is logically impossible for X to be true. In other words, X is necessarily false. Example: ‘p • ¬p’ is self-contradictory. See (1) self-contradiction and (2) self-contradictory propositional form.

**self-contradictory propositional form.** A propositional form that has only false substitution instances. A propositional form that is necessarily false, logically false, formally false, or false by virtue of its form alone. In other words, a propositional form that cannot be true. Example: ‘p • ¬p’. See (1) contingent propositional form, (2) propositional form, (3) self-consistent propositional form, (4) self-contradiction, (5) self-contradictoriness, and (6) tautologous propositional form.

**sentence.** A unit of language (i.e., a linguistic entity) that expresses a complete thought. Every sentence is in a particular language, such as English. Two sentences that have the same meaning are said to be synonymous (i.e., to bear the relation of synonymy). A sentence that has more than one meaning is said to be ambiguous (i.e., to have the property of ambiguity). Sentences can be declarative (e.g., “The Boston Red Sox won the 2013 World Series”), imperative (“Please close the door”), interrogative (“Where is the library?”), or exclamatory (“Hooray!”). See (1) ambiguity, (2) proposition, and (3) synonymy.

**simple predicate.** In predicate logic, a propositional function having some true
and some false substitution instances, each of which is an affirmative singular proposition. See (1) predicate and (2) propositional function.

**simple proposition.** In propositional logic, a proposition that does not contain any other proposition as a component. See (1) component and (2) compound proposition.

**simplification (Simp).** In propositional logic, a rule of inference (i.e., an elementary valid argument form) in which the premise is a conjunction and the conclusion is the first conjunct of that conjunction. Formally, ‘p • q’; therefore, ‘p’. See (1) elementary valid argument form and (2) inference, rules of.

**singular proposition.** A proposition that affirms or denies that a specified individual (object) has a specified attribute. Examples: “Socrates is a philosopher” (an affirmative singular proposition); “This table is not an antique” (a negative singular proposition). See (1) general proposition and (2) proposition.

**small scope.** See scope.

**“some.”** The particular affirmative and particular negative quantifier, as in (respectively) ‘Some S are P’ and ‘Some S are not P’. In logic, the word “some” means “at least one.” Thus, it does not follow from the fact that ‘Some S are P’ that ‘Some S are not P’; nor does it follow from the fact that ‘Some S are not P’ that ‘Some S are P’. See (1) “all,” (2) “no,” and (3) quantifier.

**sorites.** A chain of syllogisms in which the conclusion of one is a premise in another, in which all the conclusions except the last one are unexpressed, and in which the premises are so arranged that any two successive ones contain a common term. More generally, a chain argument in which only the premises and the final conclusion are stated, the intermediate conclusions being hidden or suppressed. See (1) chain argument and (2) syllogism.

**soundness.** A property of (some) deductive arguments. A deductive argument is sound if and only if (1) it is valid and (2) it has true premises. It follows from this definition—though it is not part of the definition—that all sound arguments have true conclusions. If the conclusion of a particular deductive argument is false, therefore, then the argument is unsound—either because it is invalid or because it has at least one false premise, or both. Soundness is the deductive analogue of inductive cogency. Soundness, like validity, is all or nothing; it is not a matter of degree. See (1) truth, (2) unsoundness, and (3) validity.

**specific form.**
1. The specific form of a given argument is the argument form that results when each different proposition of the argument is replaced with a different propositional variable. For example, the specific form of the argument ‘B ∨ G; ‘G’; therefore, ‘B’ is ‘p ∨ q; ‘q’; therefore, ‘p’, not ‘p; ‘q’; therefore, ‘r’. For any given argument, there is a unique argument form that is the specific form of that argument. An argument is valid if and only if the specific form of that argument is a valid argument form. To prove that a given argument is invalid, therefore, one must prove that the specific form of that argument is invalid. See (1) validity and (2) variable.

2. The specific form of a given proposition is the propositional form that results when each different component of the proposition is replaced with a different propositional variable. For example, the specific form of the proposition “The blind prisoner has a red hat or the blind prisoner has a white hat” is ‘p ∨ q’. See (1) component and (2) variable.

square of opposition. A diagram in the form of a square that represents various kinds of opposition (logical relations) that hold (obtain) between the four standard-form categorical propositions (“A,” “E,” “I,” and “O”). There are two squares of opposition:

- In the traditional (Aristotelian) square of opposition, all four standard-form categorical propositions have existential import. On this interpretation, the “A” proposition and its corresponding “E” proposition are contraries; the “I” proposition and its corresponding “O” proposition are subcontraries; the “A” proposition is the superaltern of its corresponding “I” proposition; the “E” proposition is the superaltern of its corresponding “O” proposition; the “A” proposition and its corresponding “O” proposition are contradictories; the “E” proposition and its corresponding “I” proposition are contradictories.

- In the modern (Boolean) square of opposition, only the “I” proposition and the “O” proposition have existential import. On this interpretation, the “A” proposition and its corresponding “O” proposition are contradictories; the “E” proposition and its corresponding “I” proposition are contradictories.

See (1) Aristotle, (2) Boole, George, (3) opposition, and (4) standard-form categorical proposition (SFCP).

standard-form categorical proposition (SFCP). There are four standard-form categorical propositions: ‘All S are P’ (known as an “A” proposition); ‘No S
are P’ (known as an “E” proposition); Some S are P’ (known as an “I” proposition); and Some S are not P’ (known as an “O” proposition). See (1) “A” proposition, (2) categorical proposition, (3) “E” proposition, (4) “I” proposition, and (5) “O” proposition.

**standard-form categorical syllogism (SFCS).** A categorical syllogism in which (i) both of its premises and its conclusion are standard-form categorical propositions and (2) the premises and the conclusion are arranged in a specified standard order. That order is: (1) major premise; (2) minor premise; (3) conclusion. Only 24 of the 256 SFCSs are valid, 15 of them unconditionally and nine conditionally. Here is a breakdown of the 24 valid SFCSs, by mood and figure, with superscripts indicating which class—subject, predicate, or middle—must have members in order for the SFCS to be valid:

<table>
<thead>
<tr>
<th>Figure</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>AEE</td>
<td>AAI”</td>
<td>AAI’</td>
<td></td>
</tr>
<tr>
<td>Barbara Canada</td>
<td>Camestres Manchester</td>
<td>Darapti</td>
<td>Bramantip</td>
<td></td>
</tr>
<tr>
<td>AAI’</td>
<td>AEO’</td>
<td>AII</td>
<td>AEE</td>
<td></td>
</tr>
<tr>
<td>Darapti</td>
<td>Falepton</td>
<td>Datisi Pacific</td>
<td>Camenes Valverde</td>
<td></td>
</tr>
<tr>
<td>AII</td>
<td>AOO</td>
<td>EAO””</td>
<td>AEO’</td>
<td></td>
</tr>
<tr>
<td>Darii Tahiti</td>
<td>Baroco Rangoon</td>
<td>Fesapo</td>
<td>Falepton</td>
<td></td>
</tr>
<tr>
<td>EAE</td>
<td>EAE</td>
<td>EIO</td>
<td>EAO””</td>
<td></td>
</tr>
<tr>
<td>Celarent New Haven</td>
<td>Cesare Seattle</td>
<td>Feronison Wellington</td>
<td>Fesapo</td>
<td></td>
</tr>
<tr>
<td>EAO”’</td>
<td>EAO’</td>
<td>IAI</td>
<td>EIO</td>
<td></td>
</tr>
<tr>
<td>Fesapo</td>
<td>Fesapo</td>
<td>Disamis Britain</td>
<td>Fresison Levittown</td>
<td></td>
</tr>
<tr>
<td>EIO</td>
<td>EIO</td>
<td>OAO</td>
<td>IAI</td>
<td></td>
</tr>
<tr>
<td>Ferio Mexico</td>
<td>Festino Leighton</td>
<td>Bocardino Monaco</td>
<td>Dimaris Miami</td>
<td></td>
</tr>
</tbody>
</table>

See (1) categorical syllogism, (2) figure, (3) mood, and (4) standard-form categorical
proposition (SFCP).

**statement.** See proposition.

**strength.** A property of (some) inductive arguments. Whether a particular inductive argument is strong (as opposed to weak) depends solely on its form or structure; it has nothing to do with the content or subject matter of the argument (i.e., what it is about). The following chart provides a number of different definitions of the term “strength” (together with its opposite, “weakness”):

<table>
<thead>
<tr>
<th>Inductive Arguments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strong</strong></td>
</tr>
<tr>
<td>It is improbable that the premises are true and the conclusion false (or: It is improbable that the conclusion is false, given that the premises are true).</td>
</tr>
<tr>
<td>The conclusion follows probably from the premises.</td>
</tr>
</tbody>
</table>

Strength is the inductive analogue of deductive validity (and weakness the inductive analogue of deductive invalidity). With respect to inductive arguments, the terms “strong” and “weak” are (1) jointly exhaustive (meaning that every inductive argument is either strong or weak); and (2) mutually exclusive (meaning that no inductive argument is both strong and weak). Strength is a matter of degree; unlike deductive validity, it is not all or nothing. Therefore, it makes sense to say, of an inductive argument, that it is “almost strong” (implying that it is not strong), “very strong” (implying that it is strong), or “somewhat strong” (implying that it is strong). It also makes sense to say that one inductive argument is “stronger” than another inductive argument (implying that both are strong). Strength is not a subjective matter, in the sense of being dependent on what some or many people believe; it is an objective feature of an inductive argument, independent of what anyone believes. It is about how the premises are related (objectively) to the conclusion. See (1) cogency and (2) validity.

**strict implication.** See logical implication.

**strict implication, paradox of.** See technical validity.

**subaltern (a.k.a. subimplicant).** See subalternation.

**subalternation (a.k.a. superimplication).** A logical relation between two propositional forms (or propositions). Propositional form X is the superaltern of
propositional form Y if and only if (1) it is logically impossible for X to be true while Y is false and (2) it is logically possible for Y to be true while X is false. In other words, (1) X logically implies Y and (2) Y does not logically imply X. Colloquially, X can’t be true while Y is false; Y can be true while X is false. Example: ‘p • q’ is the superaltern of ‘p ∨ q’. On the traditional (Aristotelian) interpretation of standard-form categorical propositions, but not on the modern (Boolean) interpretation, (1) the “A” proposition is the superaltern and the corresponding “I” proposition its subaltern, and (2) the “E” proposition is the superaltern and the corresponding “O” proposition its subaltern. See (1) logical implication and (2) standard-form categorical proposition (SFCP).

subcontraries. Two propositional forms (or propositions) are subcontraries (of one another) if and only if they stand in the logical relation of subcontrariety to one another. In categorical logic, “I” and “O” propositions are subcontraries on the traditional (Aristotelian) interpretation but not on the modern (Boolean) interpretation. See (1) contraries and (2) subcontrariety.

subcontrariety. A logical relation between two propositional forms (or propositions). Propositional form X is the subcontrary of propositional form Y (i.e., X and Y are subcontraries [of one another]) if and only if (1) it is logically possible for both X and Y to be true and (2) it is logically impossible for both X and Y to be false. Colloquially, X and Y can both be true; X and Y can’t both be false. Example: ‘p ∨ q’ is the subcontrary of ‘¬p ∨ ¬q’. See subcontraries.

subject term. In a standard-form categorical proposition, the term that comes immediately after the quantifier. See (1) predicate term and (2) standard-form categorical proposition (SFCP).

substitution instance.

1. Any argument that results from the substitution of propositions for propositional variables in an argument form. See argument form.
2. Any proposition that results from the substitution of propositions for propositional variables in a propositional form. See propositional form.

sufficient condition. The condition represented by the antecedent in a conditional. Example: In the conditional “If 4 > 3, then 4 > 2,” four’s being greater than three is a sufficient condition of four’s being greater than two. In the conditional ‘p ⊃ q’, ‘p’ is a sufficient condition of ‘q’. See (1) conditional and (2) necessary condition.
superaltern (a.k.a. principal and superimplicant). See subalternation.

superimplication. See subalternation.

syllogism. A deductive argument consisting of two premises and a conclusion. See asyllogistic inference.

syllogistic rules and fallacies (a.k.a. the rules of the syllogism). In categorical logic, a set of three rules that enable the reasoner/arguer to avoid (formal) fallacies. A set of higher-order statements laying down the conditions to which a given syllogistic inference-pattern must conform if it is to be valid. What follows are the rules for the modern (Boolean) interpretation of standard-form categorical propositions (according to which only “I” and “O” propositions have existential import):

<table>
<thead>
<tr>
<th>Rule Number</th>
<th>Statement of Rule</th>
<th>Associated Fallacy</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>The middle term must be distributed exactly once.</td>
<td>Undistributed middle</td>
</tr>
<tr>
<td>II</td>
<td>No end term may be distributed only once.</td>
<td>Illicit major; illicit minor</td>
</tr>
<tr>
<td>III</td>
<td>The number of negative premises must equal the number of negative conclusions.</td>
<td>(Unnamed)</td>
</tr>
</tbody>
</table>

If a standard-form categorical syllogism violates any of these rules, it is invalid (i.e., fallacious), whereas if it conforms to all three rules, it is valid (i.e., not fallacious). Some syllogisms violate more than one rule. At least one syllogism (OOI-3) violates all three rules. See (1) illicit process of the major term, fallacy of, (2) illicit process of the minor term, fallacy of, (3) square of opposition, and (4) undistributed middle, fallacy of.

symmetry. A relation such that if one thing has that relation to a second, then the second must have that relation to the first. Symbolically: (x)(y)(Rxy ⊃ Ryx). Examples: “is a sibling of,” “exists at the same time as,” “equals,” “is next to,” “is married to,” “has the same weight as,” “is a member of the same family as.” See (1) asymmetry, (2) nonsymmetry, and (3) relation.

synonymy. A property of linguistic entities, such as words and sentences. Two words or sentences are synonymous when they have the same meaning. Example (words): “water” and “H₂O.” Example (sentences): “John loves Mary” and “Mary is loved by John.” See (1) ambiguity and (2) sentence.
tautologousness (a.k.a. validity, logical truth, and analyticity). A logical property of propositional forms (or propositions). Propositional form X is tautologous (i.e., X is a tautology) if and only if it is logically impossible for X to be false. In other words, X is necessarily true. Example: 'p ∨ ¬p' is tautologous. See (1) tautologous propositional form and (2) tautology 1.

tautologous propositional form. A propositional form that has only true substitution instances. A propositional form that is necessarily true, logically true, formally true, or true by virtue of its form alone. In other words, a propositional form that cannot be false. Example: 'p ⊨ p'. See (1) contingent propositional form, (2) propositional form, (3) self-consistent propositional form, (4) self-contradictory propositional form, (5) tautologousness, and (6) tautology 1.

tautology.

1. (a.k.a. tautologous proposition) Any proposition whose specific form is tautologous. For example, “Either it’s raining or it’s not raining” is a tautology, since its specific form, 'p ∨ ¬p', is a tautologous propositional form. See (1) tautologousness and (2) tautologous propositional form.

2. In propositional logic, two replacement rules (abbreviated as Taut). The first rule says that a proposition is logically equivalent to a disjunction, both disjuncts of which are that very proposition. Formally, 'p' :: 'p ∨ p'. The second rule says that a proposition is logically equivalent to a conjunction, both conjuncts of which are that very proposition. Formally, 'p' :: 'p • p'. The rule allows for the elimination (or introduction) of redundancy in disjunctions and conjunctions. See replacement rules.

technical validity (a.k.a. paradox of strict implication). A property of (some) deductive arguments. An argument is technically valid (and therefore valid in the generic sense of the term) if and only if either (1) its premises are inconsistent or (2) its conclusion is a tautology. There are three ways for a set of premises to be inconsistent:

1. At least one of its members is self-contradictory.
2. Two of its members are contradictories (of one another).
3. Two of its members are contraries (of one another).

Any argument with inconsistent premises, while valid, is unsound, for sound arguments, by definition, have true premises, and inconsistent premises, by definition,
cannot all be true. Any argument with a tautologous conclusion, while valid, is uninformative, for the conclusion, being a tautology, is trivially true (e.g., “Either it’s raining here and now or it’s not raining here and now”). See (1) contradictoriness, (2) contrariety, (3) inconsistency, (4) self-contradictoriness, (5) soundness, (6) tautologousness, and (7) trivial truth.

term. See (1) predicate term and (2) subject term.

term complement. See complement.

ternary (triadic) relation. A relation that holds (obtains) between or among three individuals, i.e., a three-place relation. For example, “Texas is between New Mexico and Louisiana,” “Laban gave Zilpah to Leah,” and “The Texas Rangers traded Ian Kinsler to the Detroit Tigers.” See relation.

theoretical reasoning. Reasoning that culminates in belief (as opposed to action). The reasoning is designed to help the reasoner decide what to believe. See (1) practical reasoning and (2) reasoning.

three-dot symbol (“...”). An abbreviation for “therefore”; a conclusion indicator. See conclusion.

tilde. In propositional logic, the symbol (“~”) for negation. It appears immediately before that which is being denied. See negation.

tilde convention. In propositional logic, the convention (stipulation, agreement) that in any formula, the negation symbol will be understood to apply to the smallest proposition that the punctuation permits. Thus, the following is a disjunction with a negated first disjunct: ‘~p ∨ q’. The following is a negated disjunction: ‘~(~p ∨ q)’. See negation.

traditional (Aristotelian) square of opposition. See square of opposition.

transitivity. A relation such that if one thing has that relation to a second and the second has that relation to a third, then the first must have that relation to the third. Symbolically: (x)(y)(z)[(Rxy • Ryz) ⊃ Rxz]. Examples: “is older than,” “is north of,” “is an ancestor of,” “is a descendant of,” “weighs the same as.” See (1) intransitivity, (2) nontransitivity, and (3) relation.

transposition (Trans). In propositional logic, a replacement rule. It says that the antecedent and consequent of a conditional may be switched, provided that (1)
a tilde is added to each or (2) a tilde is subtracted from each. Formally, ‘p ⊃ q’ :: ‘¬q ⊃ ¬p’. See (i) conditional and (2) replacement rules.

**triple bar (a.k.a. tribar).** In propositional logic, the symbol ("≡") for material equivalence. See material equivalence 1.

**trivial truth (a.k.a. vacuous truth).** A proposition that is true by virtue of its form (e.g., “All dogs are dogs”) rather than by virtue of its content, matter, or substance (e.g., “All dogs are animals”). See truth.

**truth.** A property of (some) propositions. A proposition is true if and only if it correctly describes how things are. See (i) falsity and (2) proposition.

**truth condition.** In propositional logic, the truth condition of a compound proposition is the set of circumstances in which the proposition is true. For example, the truth condition of a conjunction is that both conjuncts be true; otherwise, the conjunction is false. The truth condition of a negation is that the proposition being negated is false; otherwise, the negation is false. See truth.

**truth-functional component.** In propositional logic, a component is truth-functional provided that, if it is replaced in the compound with different propositions having the same truth value as each other, the different compound propositions produced by those replacements will also have the same truth values as each other. See (i) component and (2) truth-functionality.

**truth-functional compound proposition.** In propositional logic, a compound proposition, all of whose components are truth-functional components of it. There are five truth-functional compound propositions:

<table>
<thead>
<tr>
<th>Symbol (We Will Use the First Symbol Listed)</th>
<th>Example</th>
<th>How Read</th>
<th>Name of First Symbol Listed</th>
<th>Name of Form</th>
<th>Used to Translate</th>
<th>Truth Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>¬, ¬, ~</td>
<td>~p</td>
<td>not-p</td>
<td>Tilde, Curl, Negation</td>
<td>not; it is not the</td>
<td>A negation is true iff its</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Expression</td>
<td>Interpretation</td>
<td>Notes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>----------------</td>
<td>-------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\cdot$, &amp;, $\land$</td>
<td>$p \cdot q$</td>
<td>$p$ and $q$</td>
<td>Dot Conjunction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lor$</td>
<td>$p \lor q$</td>
<td>$p$ or $q$</td>
<td>Wedge, Vee Disjunction (Alternation)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\supset$</td>
<td>$p \supset q$</td>
<td>if $p$ then $q$</td>
<td>Horse-shoe, Hook Material Implication (Conditional)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- A conjunction is true iff both its constituent propositions are true. (The constituent propositions are known as conjuncts.)
- A disjunction is true iff at least one of its constituent propositions is true. (The constituent propositions are known as disjuncts.)
- A material conditional is true iff it is not the case both that the first of its constituent propositions is true and the second false. (The first constituent proposition is known as the antecedent, and the second is known as the consequent.)
A material biconditional is true iff its constituent propositions are either both true or both false.

See (1) compound proposition and (2) truth-functional component.

**truth-functional connective.** See connective, truth-functional.

**truth-functionality.** A property of (some) compound propositions. A compound proposition is truth-functional if and only if its truth value is determined by (1) the truth values of its components and (2) the logical operators involved. Examples: The compound proposition “Abraham Lincoln was the 16th president and Andrew Johnson was the 18th president” is truth-functional. The compound proposition “Frank believes that Abraham Lincoln was the 16th president” is not truth-functional. See (1) truth-functional compound proposition and (2) truth value.

**truth-functional propositional form.** A propositional form characterized by truth-functionality. The form ‘p’ is not truth-functional. The forms ‘¬p’ and ‘p ∨ q’ are truth-functional. See (1) propositional form and (2) truth-functionality.

**truth table (a.k.a. full truth table and direct truth table).** An array (arrangement) of truth values that shows in every possible case how the truth value of a compound proposition is determined by the truth values of its simple components. Truth tables have multiple uses:

- To define truth-functional connectives, such as “¬” and “≡”.

<table>
<thead>
<tr>
<th>≡, ↔</th>
<th>p ≡ q</th>
<th>p if and only if q</th>
<th>Material Equivalence (Biconditional)</th>
<th>if and only if (iff); necessary and sufficient condition; just in case; is materially equivalent to</th>
<th>the antecedent; the second constituent proposition is known as the consequent.)</th>
</tr>
</thead>
</table>
• To classify propositions as tautologous, self-contradictory, &c.
• To compare two or more propositions as logically equivalent, contradictory, &c.
• To test certain deductive arguments for validity.

See (1) compound proposition, (2) partial truth table, and (3) truth value.

**truth value.** The attribute by which a proposition is either true or false. Every proposition, by definition, has a truth value. The truth value of a true proposition is true and the truth value of a false proposition is false. See (1) proposition and (2) truth.

**uncogency.** A property of (some) inductive arguments. An inductive argument is uncogent if and only if (1) it is weak, (2) it has a false premise, or (3) it is weak and has a false premise. Uncogency is the inductive analogue of deductive unsoundness. See (1) cogency and (2) inductive argument.

**unconditional validity.** A property of (some) deductive arguments. An unconditionally valid argument is an argument that is valid even if members of one or more classes do not exist. See (1) conditional validity, (2) deductive argument, and (3) validity.

**undistributed middle, fallacy of.** In order for a standard-form categorical syllogism to be valid, the middle term must be distributed at least once. Any syllogism that violates this rule commits the (formal) fallacy of undistributed middle. See (1) fallacy, (2) formal fallacy, and (3) standard-form categorical syllogism (SFCS).

**unit class.** A class with exactly one member. To every individual object there corresponds a unique unit class whose only member is that object itself. Socrates, for example, is the sole member of the class “things identical to Socrates.” See class.

**universal affirmative proposition.** In categorical logic, the “A” proposition, ‘All S are P’. It asserts that one class is totally included (contained) in another. See “A” proposition.

**universal generalization (UG).** In predicate logic, an operation (i.e., an elementary valid argument form) that consists of (1) introducing a universal quantifier immediately prior to a proposition, a propositional function, or another quantifier; and (2) replacing all occurrences of the constant or variable that appears in the proposition or propositional function with the variable that appears in the quantifier. Formally, ‘Øx-z’; therefore, ‘(x)Øx’. In English, this says “Anything (i.e., any
arbitrarily selected individual) is Ø; therefore, everything is Ø.” This may seem sneaky, but think of it this way. If it were true that anything one reached out and grabbed (with eyes closed) were a Ø, then it has to be the case that everything is Ø. In other words, the only way the premise could be true is if the conclusion were true as well. See (1) elementary valid argument form and (2) universal quantifier.

**universal instantiation (UI).** In predicate logic, an operation (i.e., an elementary valid argument form) that consists of removing a universal quantifier and replacing every variable bound by that quantifier with the same constant or variable. Formally, ‘(x)Øx; therefore, ‘Øa–z’. In English, this says either (1) “Everything is Ø; therefore, individual a–w is Ø” or (2) “Everything is Ø; therefore, anything (i.e., any arbitrarily selected individual) is Ø.” See (1) elementary valid argument form and (2) universal quantifier.

**universal negative proposition.** In categorical logic, the “E” proposition, ‘No S are P’. It asserts that one class is totally excluded from another. See “E” proposition.

**universal proposition.** In categorical logic, a proposition that asserts that one class is totally included (contained) in or excluded from another class. If the claim is one of inclusion, then the proposition is affirmative (‘All S are P’). If the claim is one of exclusion, then the proposition is negative (‘No S are P’). See particular proposition.

**universal quantifier.** In predicate logic, the symbol “(x),” as in “(x)Øx” (read as “Given any x, x is Ø”) or, in better English, “Everything is Ø”). The quantifier used to translate universal propositions. See (1) existential quantifier and (2) quantifier.

**universe of discourse.** The domain of reference. The things (objects) across which the quantifiers of a formal theory may range. Those objects with which a discussion is concerned. If the universe of discourse is persons, for example, then the proposition “Everything is sensitive” means “All persons are sensitive.” A universe of discourse is either restricted (to persons, sets, or lines, for example) or unrestricted. An unrestricted universe of discourse is the whole universe. To see how the universe of discourse matters in predicate logic, consider the following example. If the universe of discourse is unrestricted, then the proposition “There are people in the garden” is symbolized as (Ǝx)(Px • Gx), which is read as “There is at least one thing that is a person and is in the garden.” If the universe of discourse is restricted to persons, then the same proposition is symbolized as (Ǝx)Gx, which is read as “There is at least one person who is in the garden.” See quantifier.

**unsoundness.** A property of (some) deductive arguments. A deductive argument
is unsound if and only if (1) it is invalid, (2) it has a false premise, or (3) it is invalid \textit{and} has a false premise. \textit{See} (1) deductive argument \textit{and} (2) soundness.

\textbf{vagueness.} A property of (some) terms. A term is vague when there exist “boundary cases” such that it cannot be determined whether the term applies to them. Examples: “democracy,” “obscenity,” “death.” It may be that most terms in a natural language (such as English) are vague. One motive for creating an artificial symbolic language is to avoid the vagueness of terms in natural language. \textit{See} (1) ambiguity, (2) artificial symbolic language, \textit{and} (3) natural language.

\textbf{validity.} A property of (some) deductive arguments. Whether a particular deductive argument is valid (as opposed to invalid) depends solely on its form or structure; it has nothing to do with the content or subject matter of the argument (i.e., what it is about). The following chart provides a number of different definitions of the term “validity” (together with its opposite, “invalidity”):

\begin{table}[h]
\centering
\begin{tabular}{|l|l|}
\hline
\textbf{Deductive Arguments} & \\
\hline
\textbf{Valid} & \textbf{Invalid} \\
\hline
It is logically impossible for the premises to be true while the conclusion is false (or: It is logically impossible for the conclusion to be false given that the premises are true). & It is logically possible for the premises to be true while the conclusion is false (or: It is logically possible for the conclusion to be false given that the premises are true). \\
\hline
The truth of the premises is inconsistent with the falsity of the conclusion. & The truth of the premises is consistent with the falsity of the conclusion. \\
\hline
The conclusion follows necessarily (logically) from the premises. & The conclusion does not follow necessarily (logically) from the premises. \\
\hline
The premises, if true, provide conclusive grounds for the truth of the conclusion. & The premises, even if true, do not provide conclusive grounds for the conclusion. \\
\hline
The premises are related to the conclusion in such a way that the conclusion must be true if the premises are true. & The premises are not related to the conclusion in such a way that the conclusion must be true if the premises are true. \\
\hline
The premises logically imply the conclusion (i.e., the conclusion is logically implied by the premises). & The premises do not logically imply the conclusion (i.e., the conclusion is not logically implied by the premises). \\
\hline
The argument preserves truth. & The argument does not preserve truth. \\
\hline
\end{tabular}
\end{table}
The conditional consisting of the conjunction of the premises as its antecedent and the conclusion as its consequent is a tautology.

The conditional consisting of the conjunction of the premises as its antecedent and the conclusion as its consequent is either a self-contradiction or a contingent proposition.

The conclusion is implicit in the premises.

The conclusion is not implicit in the premises.

Validity is the deductive analogue of inductive strength. With respect to deductive arguments, the terms “valid” and “invalid” are (1) jointly exhaustive (meaning that every deductive argument is either valid or invalid); and (2) mutually exclusive (meaning that no deductive argument is both valid and invalid). Validity is all or nothing; unlike inductive strength, it is not a matter of degree. Therefore, it makes no sense to say, of a deductive argument, that it is “almost valid,” “very valid,” or “somewhat valid.” Nor does it make sense to say that one deductive argument is “more valid” (or “valider”) than another deductive argument. Validity is not a subjective matter, in the sense of being dependent on what some or many people believe (or even what everyone believes); it is an objective feature of a deductive argument, independent of what anyone believes. It is about how the premises are related (objectively) to the conclusion. See (1) deductive argument and (2) logical implication.

**variable.** In predicate logic, a symbol (lowercase “x” through “z”) that serves as a place marker, serving to indicate where the various letters “a” through “w” (constants) may be written for singular propositions to result. See (1) constant and (2) singular proposition.

**Venn, John (born 4 August 1834, Kingston upon Hull, England—died 4 April 1923, Cambridge, England).** English logician and philosopher best known as the inventor of diagrams—known as Venn diagrams—for representing categorical propositions and testing the validity of categorical syllogisms. He also made important contributions to symbolic logic (also called mathematical logic), probability theory, and the philosophy of science (from *Encyclopædia Britannica* online). See Venn diagram.

**Venn diagram.** An iconic (graphic) representation of a standard-form categorical proposition, in which spatial inclusions and exclusions correspond to the nonspatial inclusions and exclusions of classes. Venn diagrams can be used not merely to depict standard-form categorical propositions (“A,” “E,” “I,” and “O”), but to test the validity of standard-form categorical syllogisms. See (1) standard-form categorical proposition (SFCP), (2) standard-form categorical syllogism (SFCS), and (3) Venn, John.
weakness. See strength.

wedge. In propositional logic, the symbol ("\lor") for disjunction. See disjunction.

well-formed formula (WFF). In propositional logic, a compound proposition (arrangement of symbols) that is grammatically (syntactically) correct. Every formula is either well-formed or ill-formed. A given formula must obey three rules in order to be well-formed:

<table>
<thead>
<tr>
<th>Rule</th>
<th>Examples of non-WFFs</th>
<th>Examples of WFFs</th>
</tr>
</thead>
</table>
| 1. A dot, wedge, horse-shoe, or triple bar (triple bar) must go between two propositions (either simple or compound). | 1. \( p \lor q \)  
2. \( p \lor \neg(q \lor r) \)  
3. \( p \lor (q \lor r) \)  
4. \( \neg p \lor (q \lor r) \)  
5. \( p \lor q \)  
6. \( p \lor r \)  
7. \( p \lor (q \lor r) \) | 1. \( p \lor q \)  
2. \( p \lor (q \lor r) \)  
3. \( (p \equiv q) \lor (p \lor q) \) |
| 2. A tilde must be attached to the proposition (either simple or compound) it is meant to negate. | 1. \( p \lor q \)  
2. \( p \lor (q \lor r) \)  
3. \( p \lor q \lor r \) | 1. \( \neg p \)  
2. \( \neg(p \lor q) \)  
3. \( p \lor \neg q \)  
4. \( p \lor (q \lor r) \)  
5. \( \neg[p \lor (q \lor r)] \lor t \) |
| 3. Parentheses, brackets, and braces are required in order to eliminate ambiguity in a complex proposition. | 1. \( p \lor q \lor r \)  
2. \( p \lor q \lor r \) | 1. \( (p \lor q) \lor r \)  
2. \( p \lor (q \lor r) \)  
3. \( p \lor (q \lor r) \) |

See compound proposition.

\(^5\) Strictly speaking, this proposition is ambiguous (and therefore in need of punctuation). On one interpretation, it is a disjunction, the left disjunct of which is \( p \lor q \) and the right disjunct of which is \( r \). On another interpretation, it is a disjunction, the left disjunct of which is \( p \) and the right disjunct of which is \( q \lor r \). That the two interpretations have the same truth conditions does not mean that there is no ambiguity. In any event, we will treat this proposition as a non-WFF.