The *Reductio ad Absurdum*
Keith Burgess-Jackson
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One can establish (prove, demonstrate) the truth of a proposition by (1) assuming its denial (negation) and (2) validly deducing a self-contradiction (absurdity) from it, together with one or more other propositions that are known (or assumed) to be true. This procedure, or form of argument, is known by the Latin name “*reductio ad absurdum*” (reduction to absurdity). It is also known as indirect proof and *reductio ad impossibile*.

Strictly speaking, the *reductio ad absurdum* requires that all the additional propositions be known to be true. What if they’re not? What if they’re questionable or controversial? Then all one can infer from the fact that a self-contradiction has been deduced is that either the assumption or one of the additional propositions is false. One’s confidence that the assumption is false is no greater than one’s confidence that the other propositions are true. Put differently, to the extent that one is confident that the other propositions are true, one is confident that the assumption is false.

The *reductio ad absurdum* is a special case of a more general argumentative technique. Let us define “valid argument” as an argument in which, if the premises are true, then the conclusion is true. In other words, it is logically impossible for a valid argument to have true premises and a false conclusion. Valid arguments are truth-preserving. Suppose we know that the conclusion of a particular argument (call the argument “X”) is false. What else do we know? We know that either X is invalid or one or more of X’s premises is false (or both). This follows from the definition of “valid argument” just given. But all self-contradictions are false (though not conversely), so any argument the conclusion of which is a self-contradiction is either invalid or has one or more false premises. Different people may come to different conclusions about which of these is the case, and those conclusions will, in turn, require argumentation. For example, one person may claim that the assumption is false; another person may claim that one of the added propositions is false; a third person may question the validity of the argument.

Here is an example. Suppose I wish to prove that *all squirrel monkeys are primates*. I begin by assuming the denial (negation) of the proposition:

1. *Not* all squirrel monkeys are primates (i.e., *some* squirrel monkeys are *not* primates).

I then add two premises that are known to be true:

2. All monkeys are primates; and
3. All squirrel monkeys are monkeys.

From 2 and 3 I infer

4. All squirrel monkeys are primates

by a valid rule of inference. When I conjoin 4 with 1, I get the following self-contradiction:

5. All squirrel monkeys are primates and not all squirrel monkeys are primates.

Since proposition 5 cannot be true, something has gone awry. Either the argument contains an invalid inference or one of its premises (1, 2, or 3) is false. The inferences are clearly valid. Therefore, if 2 and 3 are true, then 1, which says that not all squirrel monkeys are primates, is false. But if it's false that not all squirrel monkeys are primates, then it's true that all squirrel monkeys are primates. This is the proposition that I set out to prove. Q.E.D.

A final twist. Suppose we adopt the *ad hominem* conception of argumentation, in which the aim is not to establish the truth of a proposition but to persuade a particular person to believe it. The procedure is as follows. First, state the proposition of which the interlocutor is to be persuaded. Second, assume the denial (negation) of that proposition. Third, add propositions that are already believed by the interlocutor. Fourth, using only rules of inference that the interlocutor accepts as valid, deduce a self-contradiction. This shows the interlocutor that the assumption is inconsistent with his or her beliefs. Of course, it is open to the interlocutor to reject one of the other beliefs rather than the assumption; but that just shows him or her the cost of rejecting the original proposition. (After all, you can't make people believe things.) This procedure seeks not truth, but consistency in belief.

To summarize, suppose you are trying to establish the truth of proposition p by means of a *reductio ad absurdum*. You must do the following:

1. Assume (assert) the denial (negation) of p, namely, non-p; then,
2. using only valid rules of inference (such as modus ponens),
3. deduce a known falsehood from non-p together with
4. other known-to-be-true propositions.

If you can do this, you will have shown that the assumption (non-p) is false; and if non-p is false, then p is true. This is what you set out to prove.

There are two ways to strengthen the argument. First, since someone might disagree that the deduced "falsehood" is false, try to generate a self-contradiction. In other words, show that the premises entail a necessary falsehood rather than a
contingent falsehood. Second, since someone might disagree that the “true propositions” are true, try to use only necessary truths (as opposed to contingent truths) as premises. Necessary falsehoods can’t possibly be true and can’t reasonably be affirmed (believed); necessary truths can’t possibly be false and can’t reasonably be denied (disbelieved). If the inferences are truly valid, then the only way out is to deny (disbelieve) the assumption, which is precisely what the arguer set out to do. Here, then, is the strongest possible reductio ad absurdum:

1. Assume (assert) the denial (negation) of p, namely, non-p; then,
2. using only valid rules of inference (such as modus ponens),
3. deduce a necessary falsehood (i.e., a self-contradiction) from non-p together with
4. other necessarily true propositions.

Here is the weakest possible (but still effective!) reductio ad absurdum:

1. Assume (assert) the denial (negation) of p, namely, non-p; then,
2. using only rules of inference accepted by your interlocutor as valid,
3. deduce a proposition believed by your interlocutor to be false from non-p together with
4. other propositions believed by your interlocutor to be true.

This, as I said, is the argumentum ad hominem. Do not confuse the argumentum ad hominem (argument to the person) with the ad hominem fallacy (attack on the person). The former is a legitimate argumentative technique; the latter is a fallacy. The ad hominem fallacy consists in dismissing (or finding fault with) an argument on the ground that the person making it is defective in some way. Example: Suppose you argue that capitalism is unjust and I dismiss your argument on the ground that you’re a Marxist. I have attacked you (the person), not your argument. Persons are not arguments. Good people can make bad arguments and bad people good arguments. (I’m assuming, for the sake of argument, that Marxists, as such, are bad people.)

If you want to learn more about argumentation, see Irving M. Copi and Keith Burgess-Jackson, Informal Logic, 3d ed. (Upper Saddle River, NJ: Prentice Hall, 1996).