Homework #7: Examples in Ch. 7 (paper) : Due 10/26

→ change numbers in the examples by yourself

Homework #8: Examples in Ch. 8 (paper) : Due 10/31

(1, 2, 3, 4, 6, 8, 10, 13, 14(moon-> other planet), 15)

→ change numbers in the examples by yourself

Reading assignment : Appendix B, 7-3, 7-4

Quiz # 3 : today (Ch 6 – 7)

PHYS 1443

Ch. 9 Linear momentum and collisions

1. Linear Momentum
2. Linear Momentum and Forces
3. Conservation of Momentum
4. Impulse
5. Two Dimensional Collisions
6. Center of Mass
7. CM and the Center of Gravity

(Linear) Momentum

Linear momentum : mass (m) times velocity (**v**)

Mathematically…

$$\vec{p} \equiv m\vec{v}$$

Unit ?

1. *Momentum is a vector quantity.*
2. *The heavier the object, the higher the momentum*
3. *The higher the velocity, the higher the momentum*

When you drive your car (Mustang) with a speed of 20 m/s….

p = mv = 1400 kg x 20 m/s = ?

Linear momentum is useful to solve physical problems, especially the problems involving collisions of objects.

3

Linear Momentum and Forces

What can change the (linear) momentum ?

$$\vec{p} \equiv m\vec{v}$$

$$\Delta\vec{p} = \Delta(m\vec{v}) = m(\Delta\vec{v})$$ → Need Force to change **v** !!

For a given time interval

$$\frac{\Delta\vec{p}}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a} = \sum\vec{F}$$

$$\sum\vec{F} \equiv \frac{d\vec{p}}{dt}$$

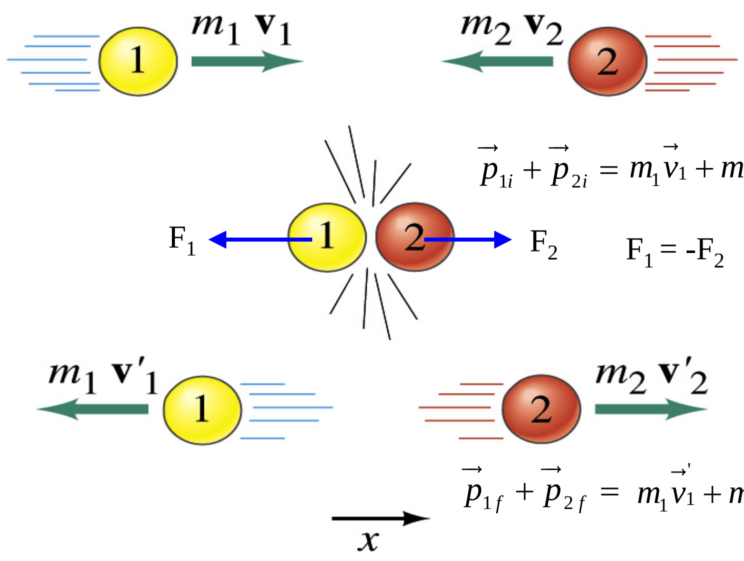
Constant mass

- The rate of the change of particle's momentum is the same as the net force exerted on it.
- When net force is 0, the particle's linear momentum is constant as a function of time. → momentum conservation !!

$$\sum\vec{F} = 0, \quad \vec{p} = cont$$

4

Linear Momentum Conservation



$$\vec{p}_{1i} + \vec{p}_{2i} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$F_1 = -F_2$$

$$\vec{p}_{1f} + \vec{p}_{2f} = m_1\vec{v}_1^{\,'} + m_2\vec{v}_2^{\,'}$$

More on Conservation of Linear Momentum in a Two Particle System

Since net force in the system is 0, total momentum of the system will conserve.

$$\sum \vec{p} = \vec{p}_2 + \vec{p}_1 = const$$

or

$$\vec{p}_{2i} + \vec{p}_{1_i} = \vec{p}_{2f} + \vec{p}_{1f}$$

or

$$\sum_{system} P_{xi} = \sum_{system} P_{xf} \quad \sum_{system} P_{yi} = \sum_{system} P_{yf} \quad \sum_{system} P_{zi} = \sum_{system} P_{zf}$$

Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.

Example for Linear Momentum Conservation

Estimate an astronaut's resulting velocity after he throws his book in the space.

From momentum conservation, we can write

$$\sum \vec{p}_i = 0 = \sum \vec{p}_f = m_A \vec{v}_A + m_B \vec{v}_B$$

Assuming the astronaut's mass is 70kg, and the book's mass is 1kg and using linear momentum conservation

$$\vec{v}_A = -\frac{m_B \vec{v}_B}{m_A} = -\frac{1}{70}\vec{v}_B$$

Now if the book gained a velocity of 20 m/s in +x-direction, the Astronaut's velocity is

$$\vec{v}_A = -\frac{1}{70}\left(20\vec{i}\right) = -0.3\vec{i}\,(m/s)$$

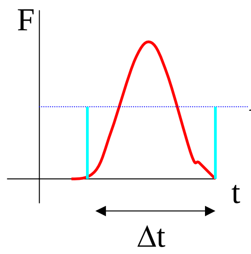
Impulse and Linear Momentum

Net force causes change of momentum ➔ *Newton's second law*

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = \vec{F}dt$$

By integrating the above equation in a time interval t_i to t_f, one can obtain impulse **J**.

$$\int_{t_i}^{t_f} d\vec{p} = \vec{p}_f - \vec{p}_i = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}dt = \vec{J}$$

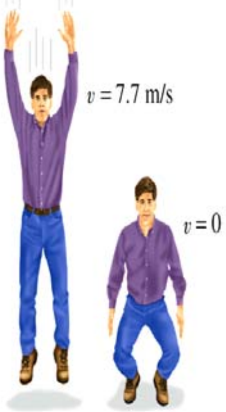


$$\overline{F} \longrightarrow \overline{\vec{F}} \equiv \frac{1}{\Delta t}\sum_i \vec{F}_i \Delta t$$

Impulse $\vec{J} \equiv \overline{\vec{F}}\Delta t$

Example 9-6

(a) Calculate the impulse experienced when a 70 kg person lands on firm ground after jumping from a height of 3.0 m. Then estimate the average force exerted on the person's feet by the ground, if the landing is (b) stiff-legged and (c) with bent legs. In the former case, assume the body moves 1.0cm during the impact, and in the second case, when the legs are bent, about 50 cm.



Obtain velocity of the person before striking the ground.

$$\Delta K = -\Delta P \qquad \frac{1}{2}mv^2 = -mg\left(y - y_i\right) = mgy_i$$

Solving the above for velocity v, we obtain

$$v = \sqrt{2gy_i} = \sqrt{2 \cdot 9.8 \cdot 3} = 7.7m/s$$

Then as the person strikes the ground, the momentum becomes 0 quickly giving the impulse

$$J = \overline{F}\Delta t = \Delta p = p_f - p_i = 0 - mv =$$
$$= -70kg \cdot 7.7m/s = -540N \cdot s$$

Example 9 – 6 cont'd

In coming to rest, the body decelerates from 7.7m/s to 0m/s in a distance d=1.0cm=0.01m.

The average speed during this period is $\overline{v} = \frac{0 + v_i}{2} = \frac{7.7}{2} = 3.8m/s$

The time period the collision lasts is $\Delta t = \frac{d}{\overline{v}} = \frac{0.01m}{3.8m/s} = 2.6 \times 10^{-3}s$

Since the magnitude of impulse is $J = \overline{F}\Delta t = 540N \cdot s$

The average force on the feet during this landing is $\overline{F} = \frac{J}{\Delta t} = \frac{540}{2.6 \times 10^{-3}} = 2.1 \times 10^5 N$

How large is this average force? $Weight = 70kg \cdot 9.8m/s^2 = 6.9 \times 10^2 N$

$$\overline{F} = 2.1 \times 10^5 N = 304 \times 6.9 \times 10^2 N = 304 \times Weight$$

For bent legged landing:

$$\Delta t = \frac{d}{\overline{v}} = \frac{0.50m}{3.8m/s} = 0.13s$$

$$\overline{F} = \frac{540}{0.13} = 4.1 \times 10^3 N = 5.9 Weight$$

Another Example for Impulse

In a crash test, an automobile of mass 1500kg collides with a wall. The initial and final velocities of the automobile are v_i= -15.0i m/s and v_f=2.60i m/s. If the collision lasts for 0.150 seconds, what would be the impulse caused by the collision and the average force exerted on the automobile?

From the problem, the initial and final momentum of the automobile before and after the collision is

Before $\vec{p}_i = m\vec{v}_i = 1500 \times (-15.0)\vec{i} = -22500\,\vec{i}\ kg \cdot m/s$

After $\vec{p}_f = m\vec{v}_f = 1500 \times (2.60)\vec{i} = 3900\,\vec{i}\ kg \cdot m/s$

Therefore the impulse on the automobile due to the collision is

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (3900 + 22500)\vec{i}\ kg \cdot m/s$$
$$= 26400\,\vec{i}\ kg \cdot m/s = 2.64 \times 10^4\,\vec{i}\ kg \cdot m/s$$

The average force exerted on the automobile during the collision is

$$\overline{\vec{F}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{2.64 \times 10^4}{0.150}$$
$$= 1.76 \times 10^5\,\vec{i}\ kg \cdot m/s^2 = 1.76 \times 10^5\,\vec{i}\ N$$

11