Projectile Motion

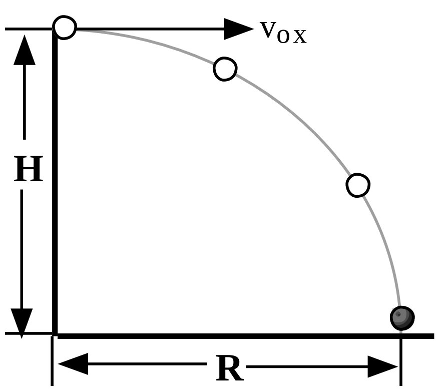
In this experiment we will study motion in two-dimensions. An object which has motion in both the X and Y direction has a two dimensional motion. We will first determine at what velocity the ball is being fired from the firing mechanism, and then with this knowledge and some calculations. Determine how far the ball will travel when it is fired at an angle other than the horizontal.

Theory:

In introductory physics courses, a projectile is an object which is given some initial velocity, v_0, and thereafter, subjected only to gravity. This definition of a projectile assumes that no force due to air resistance is acting on the projectile. This assumption is approximately valid if the velocity of the projectile is relatively small (less than 10 meters/sec) and the cross-sectional area of the object is small, which will be the case in this experiment. Since gravity is the only force assumed to act on the object **after it is given its initial velocity**, the object will be in free-fall in the vertical direction, and will move with a constant velocity in the horizontal direction.

Consider an object projected horizontally with a velocity, v_{ox}, from some initial height, H, above the floor, as sketched below. The object will travel a horizontal distance, R, during the time it falls a vertical distance, H. Since the velocity in the horizontal direction is constant,

$$R = v_{0x}\, t \qquad [1]$$



Where t is the time that the object is in flight (which is also the time it takes the object to fall a distance H).

In free fall, the vertical distance moved during a time interval, t, is given by the equation,

$$y - y_o = v_{oy}\, t - (1/2)gt^2 \qquad [2]$$

where y_o is the initial position of the object, g is the acceleration due to gravity (about 9.8 m/sec^2), and v_{oy} is the initial velocity of the object in the vertical (y) direction. In equation [2], “up” is taken as the positive direction, and “down” is

the negative direction. For the case of an object propelled horizontally, v_{oy} is zero (no component of initial velocity up or down). If the object is initially propelled from a height H above the floor, (y_o = H) then at a later time it hits the floor, and y = 0.
Thus, from equation [2],

$$-H = -(1/2)gt^2 \qquad [3]$$

and the time of flight is

$$t = \sqrt{2H/g} \qquad [4]$$

The initial velocity of the projectile can then be calculated from equation [1].

Projectile Fired at Angle θ above the Horizontal:

Consider a projectile projected with an initial velocity, v_o, at angle θ above the horizontal at height, H, above the floor, as sketched.

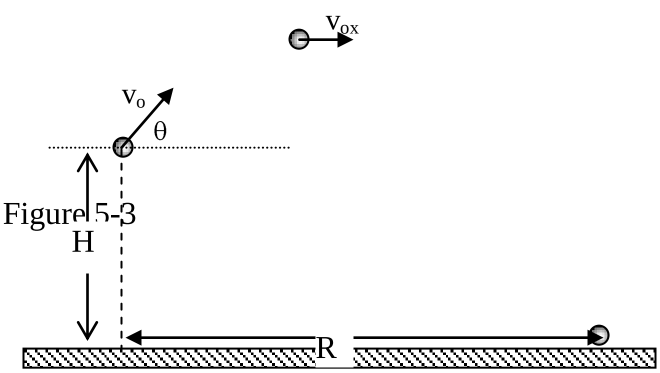


Figure 5-3

The range, R, the projectile travels can be found using kinematics equations.

First, the initial velocity v_o is broken down to its initial horizontal and vertical velocities,

$$v_{ox} = v_o\cos\theta, \qquad v_{oy} = v_o\sin\theta.$$

By rewriting equation [2] for the figure aboveit yields equation [5]. The term y_o is replaced by the term H, which is the height from the floor to the bottom of the projectile, and is shown in figure 5-3. The term v_{oy} is the initial vertical velocity of the projectile. Using the floor as the reference point the term y can be given a value of zero.

$$-1/2\ gt^2 + v_{oy}t + H = 0 \qquad [5]$$

Equation [5] is a second order polynomial and time, t, can be found using the quadratic equation.

$$0 = at^2 + bt + c$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Once a time is found the range, R, is the initial horizontal velocity, $v_{ox,}$ multiplied by the time. See equation [1].

Procedure:

1. Place the ball into the projectile launcher and use the plunger to push the ball into the mechanism until the yellow indictor is within one of the range settings. **Also be sure that the ball remains seated in the mechanism and does not roll free along the barrel.**

2. Fire the ball and take note of where it landed. Use a coin or some small object to mark the location. Create a target using a piece of carbon paper sandwiched between two sheets of computer paper. Center the target where you have marked its initial landing and tape the paper at the corners to the floor to prevent movement. Indicate on the target which edge is nearest to the launcher.

3. Measure and record the distance, d_1, in meters from the nearest edge of the paper to the small hash-mark shown on the projectile launcher that is within the depiction of the ball launching position. Measure in meters the height, H, from the floor to the bottom of the depiction of the ball launching position. (Remember the bottom of the ball hits the ground first).

4. To prevent having to chase the ball down place a wooden backdrop at the far edge of the target at an angle to the target to ricochet the ball away from the target and back towards you.

5. Reload the launcher to the same position and launch the ball. The carbon paper will make an imprint on the computer paper of where the ball struck. **There is no need to make any measurements at this time**. Repeat this until you have struck the target 5 times. Remove the paper from the floor and take it to you table. Measure the distance from the edge of the paper to each of the imprints made on the paper. Note: Some imprints may be very close together, make as good judgment as possible to the center of each imprint. Also, if all 5 imprints cannot be determined it is not necessary to repeat the process, just use the number of shots that can be determined. Record each value into the data table and find the average.

6. Find the range, R, by adding the average value from the data table to d_1. Determine the time of flight of the ball using equation [4]. Now determine the velocity of the ball using equation [1]. Record your value.

Data

	Shot 1	Shot 2	Shot 3	Shot 4	Shot 5	Ave = d_2.

d_1 = __________ d_2 = ____________

4) The range the ball travel horizontally $\mathbf{R = d_1 + d_2}$. and the height the ball fell vertically is

H = ______________ R = ____________

5) Use the equation $t = \sqrt{2H/g}$ where g = 9.8 m/s^2. to find the total time the ball was in the air. Use t, to find the velocity from the equation R = v * t.

t = ____________ v = ____________

Finding a Range for an angle other than zero degrees.

1) 1. Ask you instructor for an angle θ. Remove the projectile launcher from the ballistic pendulum. Rotate the ballistic pendulum 180°. Reconnect the projectile launcher near the top of the ballistic pendulum; the front will be secured using the single hole. The rear of the projectile launcher will be secured using the curved slot. If connected properly you should now be able to set the angle you were given by loosening the projectile launcher and lower the rear of it until the string with the plumb indicates the desired angle. From the previously determined velocity find the x and y components v_x and v_y.

   θ = _______
   $v_x = v \cos\theta$ = _________- m/s $v_y = v \sin\theta$ = __________ m/s

2. Measure in meters the height, H, from the floor to the bottom of the depiction of the ball launching position. (Remember the bottom of the ball hits the ground first).

   H = __________ m

3. To determine the time of flight for the projectile use the equation

$$y = H + v_y \sin\theta\, t - 4.9t^2$$
$$0 = c + b(t) + (-a)(t)^2$$

If we set the point of impact (the floor) as zero the y = 0 in the equation above. The quadratic equation can be used to determined t.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = _______ b = _______ c = ________ t = __________ s

4. Once the time of flight is determine, calculate the range, R, of the projectile.

   $R = v_x * t$ = __________ m

5. Measure this distance from the projectile launcher out to the floor. Place the scoring target down at the measured range position. Load the ball and when you are ready fired the ball at the target.