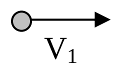
Unit 3 Help File

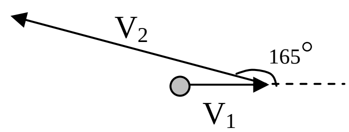
Topics
1) How to use graphical methods to determine an equilibrant.
2) How to use analytical methods to determine an equilibrant. i.e mathematically
3) Solving for two unknown vectors using simultaneous equations.
4) Addition notes for this unit.

1) How to use graphical methods to determine an equilibrant
The method describe in the manual is the easiest way when using graphical methods to find a resultant or equilibrant. following is an example for three vectors.
We have V_1 = 120@0°, V_2 = 350@165° and V_3 = 400@260°

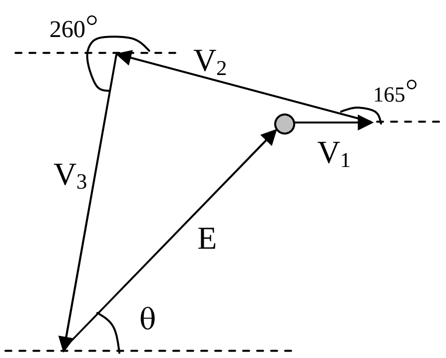
First start by selecting a scale to represent the magnitude of the vector, for example if 1cm= 100.
Next select a point of origin and then draw the first vector to scale in its indicated direction from the origin.V_1 is shown below. V_1 would be drawn 1.2 cm and the horizontal has been chosen to be zero degrees.



Next use a protractor to indicate determine the direction of the next vector V_2 (165°) in relation to the tip of the first vector. Then draw the second line to scale 3.5 cm at it given direction from the tip of V_1.



Draw in the next vector using the given direction and scale factor V_3 = 4 cm from the tip of the second vector V_2



Last connect the tip of V_3 to the start of V_1. This is the resultant or equilibrant depended on the direction of the vector. The equilibrant is shown in the diagram. Measure the length of the equilibrant using the chosen scale to find its magnitude and use a protractor to measure the angle θ.

2) How to use analytical methods to determine an equilibrant. i.e mathematically

First break each vector down into its x and y components. In this case we shall use the +x axis a zero degrees.

	mag@dir	x = mag * cos(dir)	y = mag * sin (dir)
V_1	120@0	120	0
V_2	350@165	-338.07	90.59
V_3	400@260	-69.46	-393.92
V_R		-287.53	-303.33

Note during calculation your calculator must be in Degree mode.
Sum the x values and the y values. The result is the x and y values of the resultant, R_x and R_y.

Determine the magnitude of the resultant using

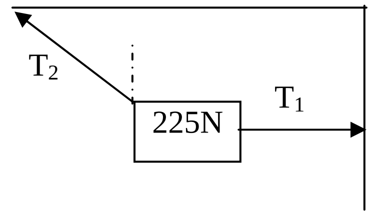
$\sqrt{R_x^2 + R_y^2} = V_R$ $\qquad \sqrt{(-287.53)^2 + (-303.33)^2} = 417.95 = 418$

The angle of V_R is determined by $\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{-303.33}{-287.53}\right) =$

When plugged into a calculator an angle of 46.5° will result, this is an incorrect angle for the resultant. The vector V_R points into the 3rd quadrant and should have an angle between 180° and 270°. Your calculator will automatically cancel out the two negatives when you have a (-x,-y) coordinate.
The resultant would have an angle of θ_R = 46.5° + 180° = 226.5°.

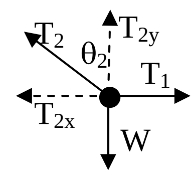
The angle of the equilibrant θ_E is in the opposite direction of θ_R and would be 226.5° -180° = 46.5°.
We now have an equilibrant of 418@46.5°. Looking at the picture on the previous page this value seems a reasonable number.

Problems 1 and 2



The figure for problems one and two is shown to the left.
And a free-body diagram for the forces is shown below it.
The body is at rest therefore the vector sum of the forces in x and y direction is equal to zero. W = 225N
Therefore

$$\Sigma F_x = T_1 – T_{2x} = 0 \quad \text{and} \quad \Sigma F_y = T_{2y} – W = 0$$



The angle θ_2 is given from the vertical.
To solve for T_2 You need to get T_{2y} in terms of T_2 with θ_2.

T_{2y} is the adjacent side and T_2 is the hypotenuse therefore

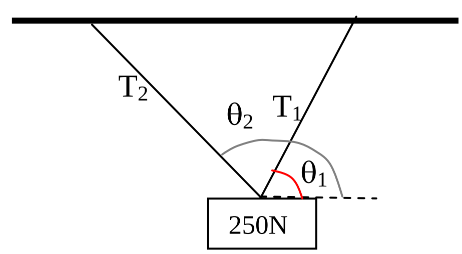
$\cos \theta_2 = T_{2y} / T_2$ Now you can solve for T_{2y}.

Then from the equation for ΣF_y also solve for T_{2y}
Knowing the weight W, the Tension in T_2 can be found by combining the two equations and solving for T_2.

To find T_1, we see that $\sin \theta_2 = T_{2x} / T_2$ here you can solve for T_{2x}.

Having found T_2 , T_{2x} can be determined and used in the equation for ΣF_x to find T_1.

3) Solving for two unknown vectors using simultaneous equations.



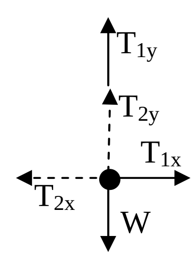
This problem is similar to problems 1 and 2 except now T_1 now has both an x and y component to its Tension.
The previous equations for the net vector forces is shown below

$$\Sigma F_x = T_1 – T_{2x} = 0 \quad \text{and} \quad \Sigma F_y = T_{2y} – W = 0$$

It is now rewritten to include the components of T_1

$$\Sigma F_x = T_{1x} + T_{2x} = 0 \quad \text{and} \quad \Sigma F_y = T_{1y} + T_{2y} – W = 0$$

To solve for the Tension T_1 and T_2
You will need to get T_{1x} , T_{2x}, T_{1y} and T_{2y} in terms of T_1 with θ_1, and T_2 with θ_2



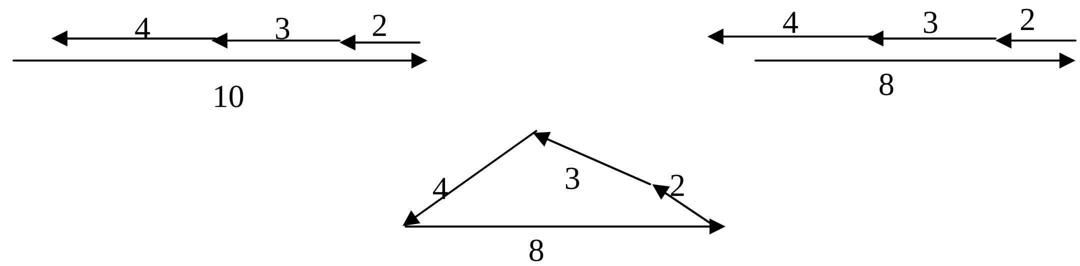
$T_{1x} = T_1 \cos \theta_1$, $T_{2x} = T_2 \cos \theta_2$ and $T_{1y} = T_1 \sin \theta_1$, $T_{2y} = T_2 \sin \theta_2$
Once this is accomplished substitute these values into the equation for
$\Sigma F_x = T_1 \cos \theta_1 + T_2 \cos \theta_2 = 0$
$\Sigma F_y. = T_1 \sin \theta_1 + T_2 \sin \theta_2 – 250 = 0$
In the equation for ΣF_x solve so that you get $T_1 = \#T_2$ where # is a value.

In the second equation substitute $\#T_2$ for T_1 and then solve for T_2.
Then return to the solution $T_1 = \#T_2$ and solve for T_1.

Questions 3 and 4

If you have a group of vectors and were free to orient them anyway you wish to achieve a given net resultant there is a simple way to test if the given group can achieve the desired net resultant .

For instances when you have greater than two vectors.
Say for example the net resultant should be zero.
Take the greatest value vector in the group and subtract each of the other vectors in the group from it.
If the result of the subtraction is greater than zero the group cannot achieve the desire net resultant of zero.
A set of vectors {2,3,4,10} would not work but {2,3,4,8} would



In the first example the shortest path back to the origin cannot be achieved therefore those vectors cannot be reposition to reach zero but they can be in the second example.

If there are only two Vectors in the set then setting up a boundary condition may be easier.

Say the desire resultant is R.
and you have a set of two values {L,G} where L is less than G

Then setting up this statement

$$(G - L) \leq R \leq (G + L)$$

If this statement is true then the resultant can be achieved.