# PHYS 1444 – Section 002
# Lecture #4
# Chapter 22

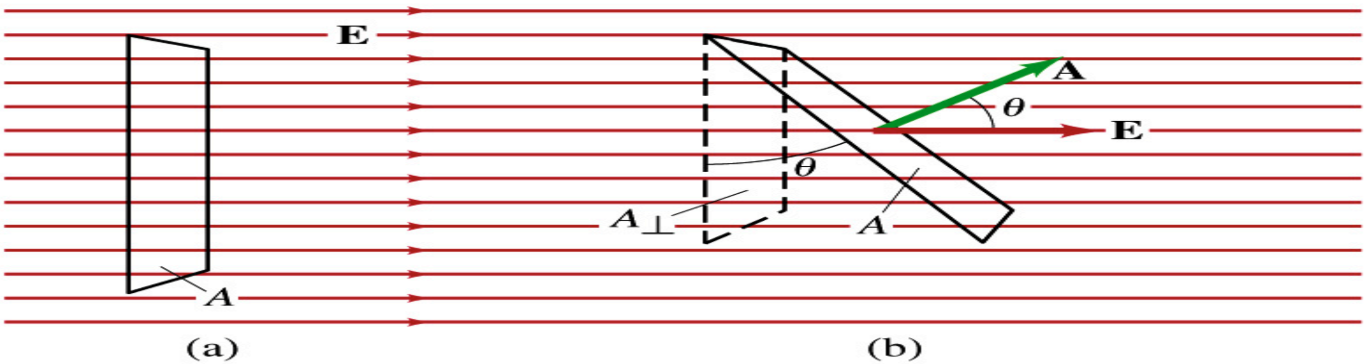
*Dr. Koymen*

- Gauss' Law
- Electric Flux
- Generalization of Electric Flux
- How are Gauss' Law and Coulomb's Law Related?
- Electric Potential Energy
- Electric Potential

# Gauss' Law

- Gauss' law states the relationship between electric charge and electric field.
  - More general and elegant form of Coulomb's law.
- The electric field by the distribution of charges can be obtained using Coulomb's law by summing (or integrating) over the charge distributions.
- Gauss' law, however, gives an additional insight into the nature of electrostatic field and a more general relationship between the charge and the field

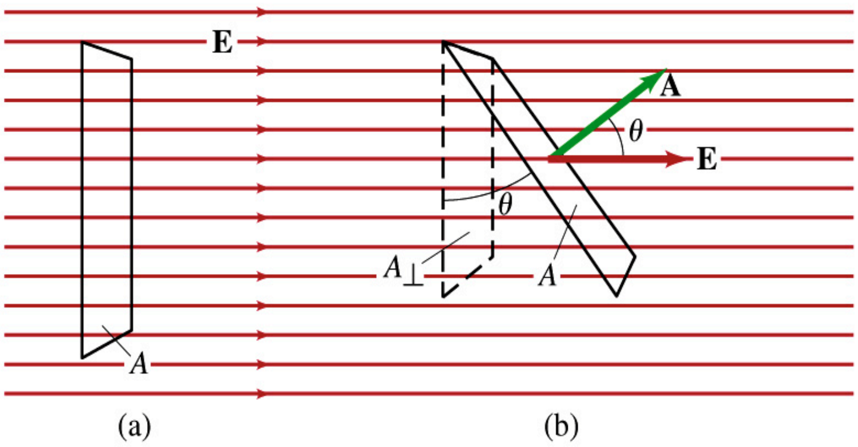
# Electric Flux




- Let's imagine a surface of area A through which a uniform electric field E passes
- The electric flux is defined as
  - $\Phi_E$=EA, if the field is perpendicular to the surface
  - $\Phi_E$=EAcosθ, if the field makes an angle θ to the surface
- So the electric flux is defined as $\Phi_E = \vec{E} \cdot \vec{A}$.
- How would you define the electric flux in words?
  - Total number of field lines passing through the unit area perpendicular to the field. $N_E \propto EA_\perp = \Phi_E$

# Example 22 – 1

- **Electric flux.** (a) Calculate the electric flux through the rectangle in the figure (a). The rectangle is 10cm by 20cm and the electric field is uniform with magnitude 200N/C. (b) What is the flux in figure if the angle is 30 degrees?




The electric flux is

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta$$

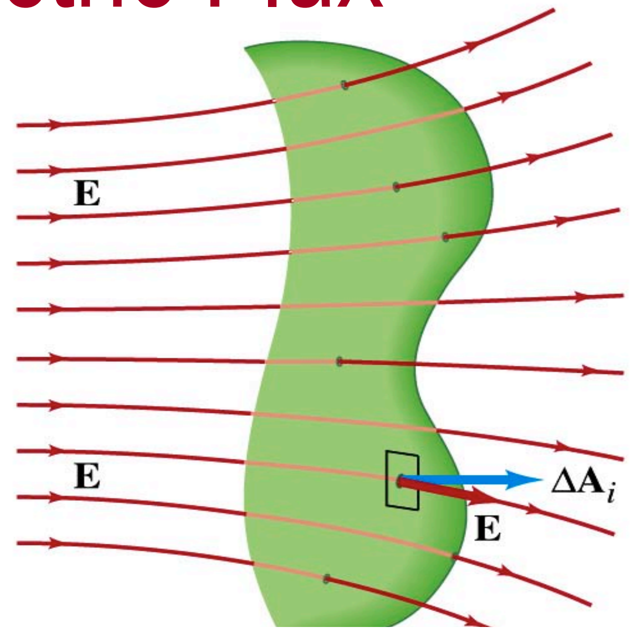
So when (a) θ=0, we obtain

$$\Phi_E = EA\cos\theta = EA = (200N/C)\cdot\left(0.1\times 0.2m^2\right) = 4.0\,\mathrm{N}\cdot\mathrm{m}^2/C$$

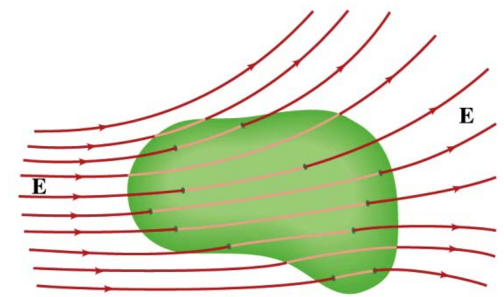
And when (b) θ=30 degrees, we obtain

$$\Phi_E = EA\cos 30^\circ = (200N/C)\cdot\left(0.1\times 0.2m^2\right)\cos 30^\circ = 3.5\,\mathrm{N}\cdot\mathrm{m}^2/C$$

# Generalization of the Electric Flux




- Let's consider a surface of area A that is not a square or flat but in some random shape, and that the field is not uniform.
- The surface can be divided up into infinitesimally small areas of $\Delta A_i$ that can be considered flat.
- And the electric field through this area can be considered uniform since the area is very small.
- Then the electric flux through the entire surface can is approximately



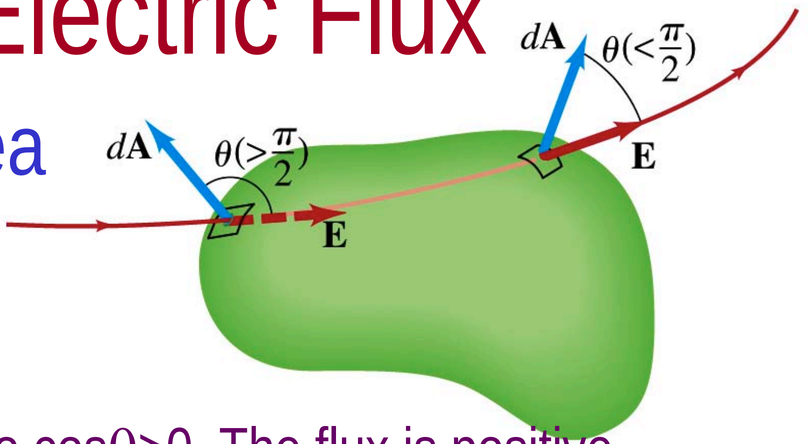

$$\Phi_E \approx \sum_{i=1}^{n} \vec{E}_i \cdot \Delta\vec{A}_i$$

- In the limit where $\Delta A_i \rightarrow 0$, the discrete summation becomes an integral.

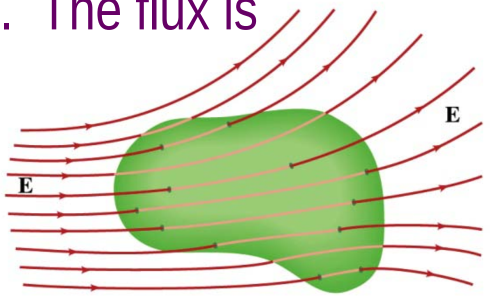
$$\Phi_E = \int \vec{E}_i \cdot d\vec{A}$$ open surface

$$\Phi_E = \oint \vec{E}_i \cdot d\vec{A}$$ enclosed surface

# Generalization of the Electric Flux



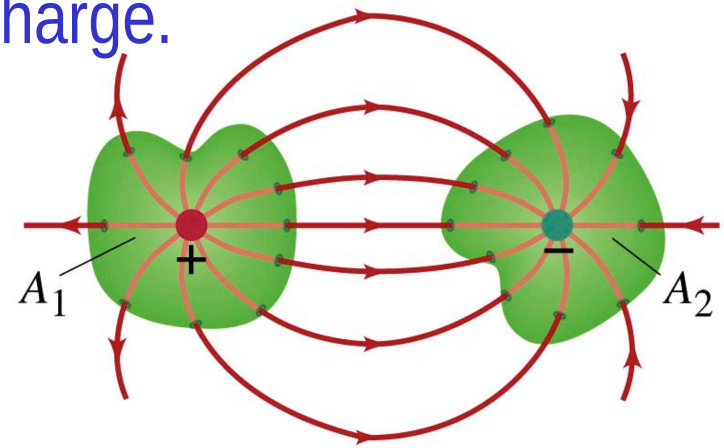

- We arbitrarily define that the area vector points outward from the enclosed volume.
  - For the line leaving the volume, $\theta<\pi/2$, so $\cos\theta>0$. The flux is positive.
  - For the line coming into the volume, $\theta>\pi/2$, so $\cos\theta<0$. The flux is negative.
  - If $\Phi_E>0$, there is a net flux out of the volume.
  - If $\Phi_E<0$, there is flux into the volume.

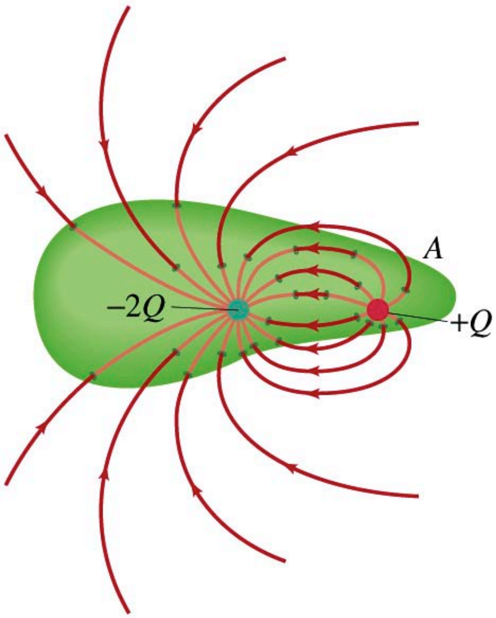



- In the above figures, each field that enters the volume also leaves the volume, so $\Phi_E = \oint \vec{E}\cdot d\vec{A} = 0.$
- The flux is non-zero only if one or more lines start or end inside the surface.

# Generalization of the Electric Flux

- The field line starts or ends only on a charge.
- Sign of the net flux on the surface $A_1$?
  - The net outward flux (positive flux)
- How about $A_2$?
  - Net inward flux (negative flux)
- What is the flux in the bottom figure?
  - There should be a net inward flux (negative flux) since the total charge inside the volume is negative.
- The flux that crosses an enclosed surface is proportional to the total charge inside the surface. ➔ This is the crux of Gauss' law.

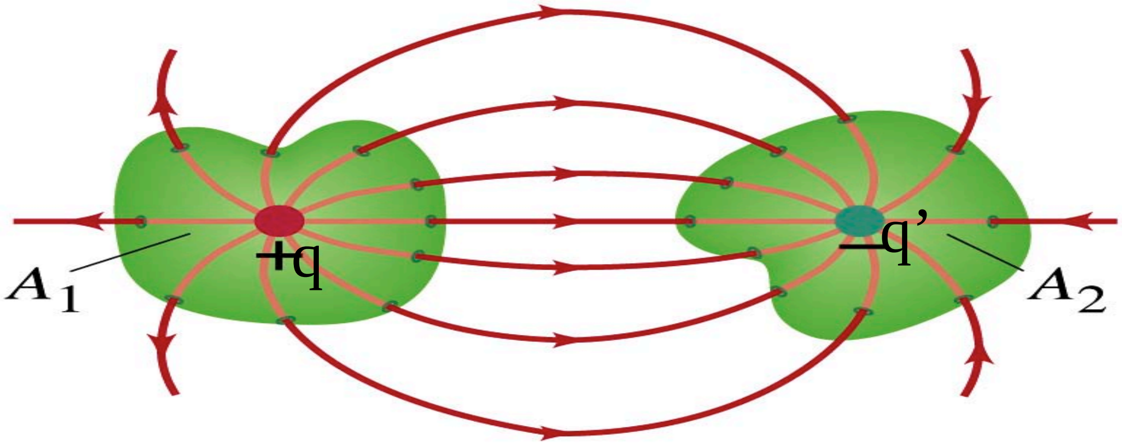


# Gauss' Law

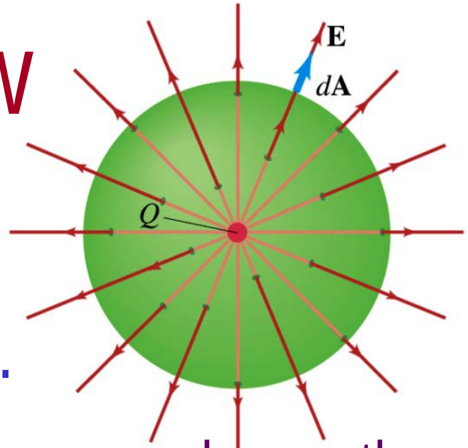
- The precise relation between flux and the enclosed charge is given by Gauss' Law $$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$
  - $\varepsilon_0$ is the permittivity of free space in the Coulomb's law
- A few important points on Gauss' Law
  - Freedom to choose!!
    - The integral is performed over the value of E on a closed surface of our choice in any given situation.
  - Test of existence of electrical charge!!
    - The charge $Q_{encl}$ is the net charge enclosed by the arbitrary closed surface of our choice.
  - Universality of the law!
    - It does NOT matter where or how much charge is distributed inside the surface.
  - The charge outside the surface does not contribute to $Q_{encl}$. Why?
    - The charge outside the surface might impact field lines but not the total number of lines entering or leaving the surface

# Gauss' Law




- Let's consider the case in the above figure.
- What are the results of the closed integral of the gaussian surfaces $A_1$ and $A_2$?
  - For $A_1$ $\oint \vec{E} \cdot d\vec{A} = \frac{+q}{\varepsilon_0}$
  - For $A_2$ $\oint \vec{E} \cdot d\vec{A} = \frac{-q'}{\varepsilon_0}$

# Coulomb's Law from Gauss' Law




- Let's consider a charge Q enclosed inside our imaginary Gaussian surface of sphere of radius r.
  - Since we can choose any surface enclosing the charge, we choose the simplest possible one! ☺
- The surface is symmetric about the charge.
  - What does this tell us about the field E?
    - Must have the same magnitude at any point on the surface
    - Points radially outward / inward parallel to the surface vector dA.
- The Gaussian integral can be written as

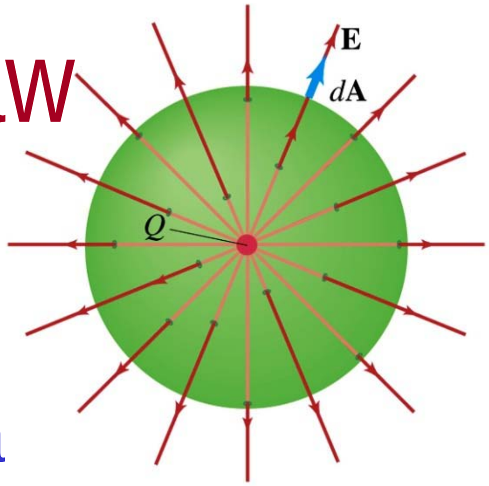
$$\oint \vec{E} \cdot d\vec{A} = \oint E\,dA = E \oint dA = E\left(4\pi r^2\right) = \frac{Q_{encl}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

Solve for E

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Electric Field of Coulomb's Law

# Gauss' Law from Coulomb's Law




- Let's consider a single static point charge Q surrounded by an imaginary spherical surface.
- Coulomb's law tells us that the electric field at a spherical surface is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

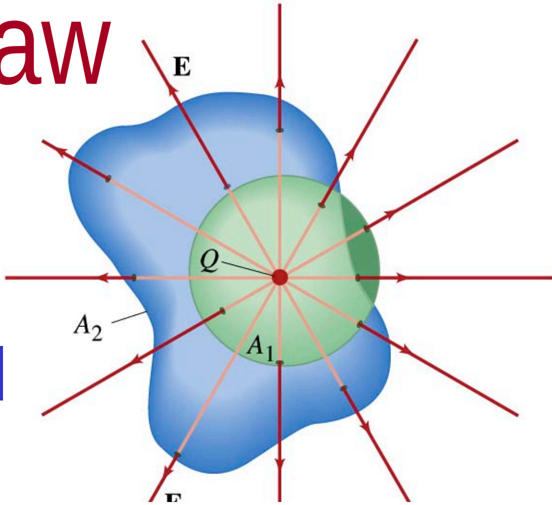
- Performing a closed integral over the surface, we obtain

$$\oint \vec{E}\cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}\cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dA$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\oint dA = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}$$

Gauss' Law

# Gauss' Law from Coulomb's Law Irregular Surface




- Let's consider the same single static point charge Q surrounded by a symmetric spherical surface $A_1$ and a randomly shaped surface $A_2$.
- What is the difference in the number of field lines passing through the two surface due to the charge Q?
  - None. What does this mean?
    - The total number of field lines passing through the surface is the same no matter what the shape of the enclosed surface is.
  - So we can write: $\oint_{A_1} \vec{E}\cdot d\vec{A} = \oint_{A_2} \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$
  - What does this mean?
    - The flux due to the given enclosed charge is the same no matter what the shape of the surface enclosing it is. ➔ Gauss' law, $\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$, is valid for any surface surrounding a single point charge Q.